



Electric Power Bid Determination and Evaluation for Price Taker Units under Price Uncertainty

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ABSTRACT

Power companies aim to maximize their profit which is highly related to the bidding strategies used. In order to sell electricity at high prices and maximize their profit, power companies need suitable bidding models that consider power operating constraints and price uncertainty within the market. Price taker units have no power to affect the prices but need to determine their best bidding strategy to maximize their profit assuming a quadratic cost function and uncertain market prices. Price taker units also need to evaluate their bidding strategy under different price scenarios. In this paper, we first model the bidding problem for a price taker unit and then propose quadratic programming, nonlinear programming and marginal cost based bidding models under price uncertainty. We use case studies to study the computation burden and limitation to reach a solution. We also propose a simulation methodology to evaluate the performance of each bidding strategy for different market prices in an effort to help decision makers to assess their bidding decisions.

Keywords: Bidding, Nonlinear Programming, Quadratic Programming, Simulation, Electricity Markets

JEL Classifications: L94, D44, D47, E17

1. INTRODUCTION

Electricity is generally accepted as different from other commodities. It is still not storable economically, and its demand is instantaneous, so it must be produced and used in real time while the demand is continuous. These unique characteristics of electricity and the necessity of real time balance create a need for coordinated markets in which power plants, transmission grid, and distribution lines have to be in a close but well-defined relationship. The price is strongly load dependent, highly volatile, seasonal and consumption dependent. The elasticity of electricity demand to price is low as electricity is a unique commodity, and it is difficult to replace it. Also, small consumers are not affected by instant price changes as a utility company usually provides their electricity. The parameters are stochastic, which gives a stochastic behaviour to the electricity price. Energy consumption, fuel costs, availability of fuels, equipment capacity and market participants'

behaviour are stochastic and unknown to other players. The market prices are set based on the economic principles where all sellers' agree to deliver the cumulative demand and all buyers' agree to buy the offered quantity at a determined price level. The bilateral energy markets have their own structure; and the price is unique in that it is determined between the buyer and the seller. In poolco and power exchanges, on the other hand, the buyers and sellers receive the market price determined in auctions after the buy and sell offers are submitted.

The main objective of the market is to provide a perfect competitive environment in which an Independent System Operator (ISO) is responsible for the coordination of physical operations that include scheduling the generation, making sure they continue the balance of supply and demand, supporting services for reliability, and coordination with other markets. ISO oversees the system operation; determines the transmission schedule; and has the right

to take measures against players that do not respect the generation or the consumption plan. Perfect competition and oligopoly are two models of interest in the deregulation of the electricity market. Thermal plants such as coal and gas fired units have nonlinear cost functions; and their marginal costs are related to the quantity of produced electricity. In practice, a deregulated market is not considered perfectly competitive due to the limited number of suppliers. A supplier tends to bid higher than its marginal cost as a solution to Strategic Bidding Problem (SBP) (David and Wen, 2000).

A power supplier or producer aims to maximize its profit and decrease its risks, and he needs to submit bids to the market considering its constraints and market conditions. The markets that the bids are submitted to can be classified as day-ahead, hour-ahead, real time; and the reserve market in which the actual time remaining for operation differs. A submitted bid might be accepted if the price is lower than the Market Clearing Price (MCP), and the offer will be cleared with the market price. On the other hand, if the offered bid is above the hourly MCP, then the offer will not be selected; and there will be no revenues. Assuming a uniform bidding mechanism, all bidders will be paid with MCP as it is important to be in accepted bids for the price taker units since they do not have the power to affect the market price. On the other hand, for the pay as bid (PAB) type mechanism, the bidder will receive what he has offered if his bid is accepted. Then, the main objective of SBP is to determine the proper price and quantities for the power that will be submitted to the market. Knowledge about MCP is the most important parameter for the decision making process about bidding blocks. A block consists of a price and corresponding power quantity. If a player is a market maker, then he can affect the market price using his power capacity as in oligopoly models such as Cournot (Kian et al., 2005) or the supply function equilibrium (Rajaraman and Alvarado, 2003). These models need the cost data of the market players, which are often not available. They also face some issues such as lack of equilibrium or the existence of multiple equilibriums. On the other hand, if the future values of MCP can be accepted as a random variable like in (Valenzuela and Mazumdar, 2003) and (Yucekaya et al., 2009), MCP can be considered exogenous, and can be included in the decision making process (Mazzi et al., 2017). Such an approach is even more suitable for price taker units as they do not have the power to affect the MCP. It is shown that generating units can be considered separately if the price is assumed exogenous (Valenzuela and Mazumdar, 2003). When analysing a price taker and other players in a competitive market, the behaviours of other players are ignored, and the problem of the price taker player is simplified. Such an approach requires the price taker player to predict the final price of the market according to which he will take an action. If the interactions need to be analyzed, game theory is commonly used to analyze the behaviours of the players and finding the equilibrium. Agent based simulation models also provide a framework to reach the equilibrium; and they are commonly used when the complexity of the problem increases (Yucekaya and Valenzuela, 2013). These models also help to observe the interaction between players when they aim to maximize their individual objective function until all of them reach an equilibrium. As the day-ahead market is repeated daily,

the players are able to learn the rules and observe the consequences of their strategies and reactions of other players. The market prices might converge to stable distributions for off-peak periods or when the demand forecast is simplified. On the other hand, repetition also gives suppliers the option to change their bidding strategies if an opportunity arises as a result of factors such as transmission congestion, higher demand, and rule change (Mathur et al., 2017a).

There are many efforts in the literature to analyse the bidding mechanism in the markets. The bidding strategies used in the market are discussed in (David and Wen, 2000), (Prabavathi and Gnanadass, 2015) and (Mathur et al. 2017b). They present a detailed review of the bidding strategies in competitive markets.

As bidding problem has dynamic interaction and market operations, both optimization and heuristic approaches are used to model and solve the bidding problem for market players. Optimal control (Liu and Wu, 2006), game theory (Song et al., 2003) and (Kian and Cruz, 2005), Lagrangian relaxation (Zhang et al., 2000), dynamic programming (Jiang and Powell, 2015), bilevel programming and swarm (Zhang et al., 2010), information gap decision theory (Nojavan et al., 2015), Shuffled Frog Leaping Algorithm (Kumar and Kumar, 2014), point estimate method (Peik-Herfeh et al., 2013), stochastic cournot model (Sharma et al., 2014), stochastic optimization (Davatgaran et al., 2018), and (Song and Amelin, 2017), and simulation (Yucekaya, 2013) are some of the recent research areas on the bidding problem. It is also possible to use hybrid models and include operational characteristics to model and solve the bidding problem. Senthilvadivu et al. (2019) propose a hybrid technique that includes recurrent neural network, support vector machine, and the lightning search algorithm to develop bidding strategies aiming to reach maximum profit for suppliers and consumers. Nazari and Ardehali (2019) propose a bidding strategy development method in day-ahead and spinning reserve markets considering emission and wind, pumped storage, and thermal system.

The research for price maker and price taker bidding strategy need to be separated (Sadeghi-Mobarakeh and Mohsenian-Rad, 2016). Song and Amelin (2017) develop a bidding strategy for a price-maker retailer with flexible demands including the risk levels. Kohansal and Mohsenian-Rad (2015) develop a stochastic optimization framework to determine bid and a corresponding quantity for the market. Such studies need to analyse the impact of their bids on the market price. There are some studies that only focus on developing price taker bidding approaches such as Conejo et al. (2002); De Ladurantaye et al. (2007); and Fleten and Pettersen (2005). Mazzi et al. (2017) propose a stochastic optimization method for a price taker unit that is bidding in a two-settlement way, and PAB electricity market. They generate electricity market prices and use scenarios for the day-ahead and balancing market. Mathur et al. (2017) propose a genetic algorithm based method for price taker units in which they consider both symmetric and asymmetric information for the decision making process.

In this paper, as a contribution to literature, we model the SBP for price taker electric power generators, and find a solution

using quadratic and nonlinear programming given that the power producer has imperfect price estimations. However, an optimal solution can only be obtained within a reasonable computational time for a limited number of price scenarios as the computational time increases exponentially as the size of the problem increases. We also propose a marginal cost based bidding methodology where a power producer can submit its marginal cost as a bid. It is worth mentioning that the prices are accepted as exogenous and a market player has no perfect information for the upcoming prices. Hence, the offered bids need to be evaluated for different price scenarios. In order to evaluate a bidding strategy for any given market price scenarios, we propose a spreadsheet based simulation algorithm to evaluate bids and help companies with their decision analysis.

The remainder of this paper is organized as follows. Section 2 provides a description of the market design and bidding mechanism. The formulation of the problem with different methods is introduced in Section 3. Section 4 provides a case study for different price scenarios in an effort to measure the performance of the proposed methods. Finally, in Section 5, the concluding remarks are provided.

2. THE MARKET DESIGN AND BIDDING

2.1. The PJM Market Design

Federal Energy Regulatory Commission in the USA proposed a Standard Market Design (SMD) concept in 2002 for the standardization of electric power markets in the USA. This design and its variants are adopted by many other markets in different regions (Cramton, 2017). The objective of a typical SMD is to develop a market structure that brings together the physical system and the financial operations. This is achieved by defining the roles and the interaction of system components. SMD also deals with the system governance, market operations, risk management, market monitoring and conflict resolutions that might occur among the members. The PJM interconnection is a federally regulated and non-profit organization that manages the transmission of wholesale electricity in 13 states involving more than 65 million people. PJM's members include power generators, transmission owners, electricity distributors, power marketers, and large consumers. PJM assumed its ISO position in 1996, and introduced bid based pricing and locational marginal while it acts independently in managing the wholesale electricity market.

SMD aims to increase competition; hence, it is a good place where suppliers and consumers meet under the supervision of an ISO and economic fundamentals. The balancing of supply and demand is always crucial in an economic market. However, it is vital for an electricity market since the lack of electricity when needed can lead to very costly consequences.

2.2. Bidding in PJM Power Market

The market players need to submit bids for both buying and selling the power in the day ahead market. An offer includes at most ten price and corresponding quantity pairs. These blocks need to be submitted to the day ahead market until noon before the actual operation day. The players might estimate the hourly MCP's but need not communicate with each other, and need to keep their

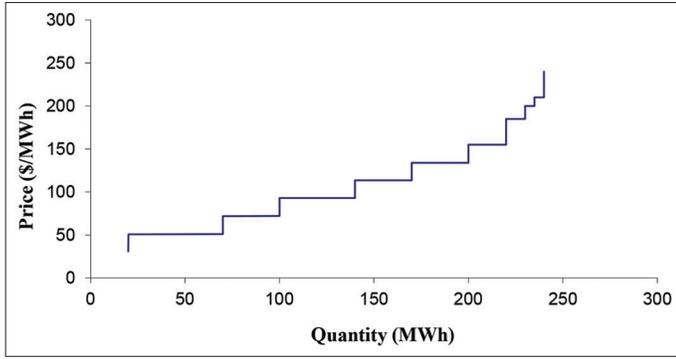
offers and cost data as a secret. SMDs usually use uniform price auctions and PAB auctions to govern the market mechanism. After the bids are submitted, and the market is settled by the system operator, all the dispatched generators in the uniform price auction are paid the market price whereas they got paid their bid price in PAB auction. The selection process for winning generators and the equilibrium price are the same for both designs with the difference that the generators would make different revenues. A supplier or generator expects to maximize its profit once its generation cost is deducted. When the player is a price taker unit, his first objective is to be selected as a dispatched unit; and for that, he needs to submit a price lower than the MCP. On the other hand, if the MCP is lower than his marginal cost he might be making a loss instead of a profit if he submits a price lower than his marginal cost. PJM also limits the offered prices with a price cap. The day ahead bids are financially binding commitments; and the day ahead prices remain fixed for all transactions scheduled in the day ahead market. The deviations from the day ahead prices are expected; and the real time prices are used to price these deviations. On the other hand, if a generator bids into the market, and fails to deliver as scheduled, he is still liable for the quantity for which he will be charged at the real time market price.

A generator offer for the PJM market is composed of two components: the price and quantity of electricity that a supplier is willing to generate. Offers are submitted in blocks of price quantity pairs. PJM allows submitting at the most ten blocks for a generator offer. Figure 1 illustrates a valid offer curve in PJM power market. Each generating unit also submits its minimum run time, minimum down time, no-load costs and start-up costs to the PJM market. PJM runs the "two-settlement" software to determine the hourly commitment schedules and the LMPs. Generating units that have minimum run times that exceed 24 h are asked by PJM to submit binding offer prices for the next 7 h.

3. THE PROBLEM FORMULATION AND SOLUTION APPROACH

The suppliers and buyers bid into the market in an effort to maximize their objective, which is to maximize its profit and minimize its cost, respectively. If the supplier has a capacity enough to impact the price, then he might manipulate the market with his actions. On the other hand, if the supplier has a limited capacity, he has to follow the market flow and accept the MCP. We assume a thermal power generator, which obtains its revenue by selling its power to PJM market. Such a generator has no power to affect MCP in the day-ahead market, and is willing to accept the hourly price to generate committed quantity (Yucekaya et al., 2009). SBP then can be modeled for a price taker unit in which its decisions do not affect the market prices. In order to submit to the day ahead market, N price-quantity blocks at the most need to be determined each day considering the capacity and estimated market prices. Given that the purpose of this paper is to analyze and test the proposed models, we exploit a fundamental model for generating market prices, instead of using real market data. MCP values at each hour are assumed as random variables whose probability distribution has known parameters, and they are fed to the model as exogenous values.

Figure 1: A generator's offer curve in the PJM day-ahead market



The bids are valid for the day ahead market for the next day, and this process is repeated each day. For such a generator, there are N pairs of decision variables b_i and Δq_i need to be determined. The variable Δq_i denotes the amount of energy increase in block i to get the bid price b_i for delivery at any hour of the next day which are represented by the vectors Δq and b , respectively. As the SMD assumes uniform bidding, if the MCP at hour t is equal to or higher than the b_i , then the last offer at this price or lower are accepted and got paid by MCP. Thus, the total energy to be produced at time t and sold to the market at a price P_t is given by:

$$q_t = \sum_{i=1}^{I(P_t)} \Delta q_i, \text{ where } I(P_t) = \text{Max } j \text{ such that } b_j \leq P_t \text{ for } t=1..T; i=1..I(P_t) \quad (1)$$

The profit for the day-ahead market for a 24-h period can be assumed as the total revenue gained from power sales at each hour, and the cost of generation is deducted for the generated power quantities. Note that the generator makes a revenue if it generated power during hour t at the market price, P_t (\$/MWh). As P_t is a random variable, then the total profit over a period of T hours is also a random variable. For the cases where there are price scenarios, K samples of the hourly prices which have an equal probability of occurrence can be assumed. Then, the objective becomes maximizing the expected profit over the time period T (usually 24 h) considering prices at each hour t of sample k as P_t^k and generated power of at each hour t of sample k as q_t^k . Under the light of these assumptions, the bidding problem $P(\Delta q, b)$ can be represented as follows:

$$P(\mathbf{q}, \mathbf{b}) = \text{Max}_{\mathbf{q}, \mathbf{b}} E[\text{Profit}] = \frac{1}{K} \sum_{k=1}^K \sum_{t=1}^T [P_t^k q_t^k - C(q_t^k)] \text{ for } k=1..K; t=1..T; \quad (2)$$

There are some constraints that are related to market conditions, and generator operations as stated below. A bid price is limited with the price cap as in Eq. 3. A bid quantity increase and total commitment cannot be above the generator capacity (Eq. 4 and Eq. 5). A bid is selected only if the bid price is lower than MCP (Eq. 6). The cost of the energy produced by the generating unit depends on the amount of fuel consumed and is typically approximated by a quadratic cost function (Eq. 7). The coefficient a_1 represents the fixed cost or no-load cost for each hour. The value a_2 represents the linear cost which is proportional to the amount of

power produced. The parameter a_3 is the quadratic cost coefficient, and it is related to the amount of fuel used to produce electricity.

$$0 < b_i < B^{\text{max}} \text{ for } i=1..N. \quad (3)$$

$$\sum_{i=1}^N \Delta q_i \leq Q^{\text{max}} \quad (4)$$

$$0 < \Delta q_i < Q^{\text{max}} \text{ for } i=1..N. \quad (5)$$

$$q_t^k = \sum_{i=1}^{I(P_t^k)} \Delta q_i, \text{ } I(P_t^k) = \text{Max } i \text{ such that } b_i \leq P_t^k \text{ for } k=1..K; t=1..T; i=1..I(P_t^k) \quad (6)$$

$$C(q_t^k) = a_1 + a_2 q_t^k + a_3 (q_t^k)^2 \text{ for } k=1..K; t=1..T. \quad (7)$$

3.1. Quadratic Programming Model

The supplier aims to maximize its expected profit considering that the market prices are uncertain and he has constraints related to generation. Given that the submitted bidding strategy is valid for 24 h, and there are 24 hourly prices, one might reach a solution by finding a bid price for each hour. However, the number of price-quantity blocks N is limited to 10; and then at most 10 pairs of price and quantity pairs need to be determined and submitted to the market.

Quadratic Programming (QP) is one way to find an optimal solution to the bidding problem. However, it can solve relatively small-sized problems as the solution space gets larger when the number of prices increases. By setting the number of samples to one, and the number of maximum bidding blocks equal to the number of hours of the time horizon, a quadratic programming model can be formulated. Notice that when the market price consists of one sample and the number of blocks are equal to the number of hours, the optimal bidding price of a block of power is equal to one of the market prices. Therefore, the bidding problem in Eq. 2, 4, and 6 is reduced to the following mathematical representation:

$$\text{Max } Z = \sum_{t=1}^T [P_t q_t - a_2 q_t - a_3 q_t^2] \text{ for } t=1..,N \quad (8)$$

Subject to the following constraints:

$$\sum_{i=1}^N \Delta q_i \leq Q^{\text{max}} \text{ for } i=1..,N \quad (9)$$

$$q_t = \sum_{i=1}^t \Delta q_i \text{ for } t=1..,N \quad (10)$$

$$\Delta q_i \geq 0 \text{ for } i=1..,N \quad (11)$$

As the objective function has a polynomial component, and the constraints are linear, a solution for at most 10 hourly prices can be found by using such an approach. However, if the supplier has more price scenarios than he expects, it will not be possible to include all samples in this method.

3.2. Nonlinear Programming Model

It is still possible to force the bid prices to be in close proximity of the expected hourly prices, and also include more samples if additional constraints are added. Nonlinear Programming (NLP)

is the process of solving a problem that includes equalities, inequalities, constraints, and an objective function some of which is nonlinear. The proposed problem has nonlinear equalities which make the computation time larger than expected. The process finds a set of unknown real variables that makes the objective function maximized or minimized. The market prices are unknown when the bidding decision is made; hence the problem should be developed for scenarios instead of only one price sample. Having the same problem as in Eq. 2, the NLP will have the following constraints:

$$Mz_t^k(i) \geq 1.001P_t^k - b_i \quad \text{for } i = 1..N; t = 1..T; k = 1..K. \quad (12)$$

$$M(z_t^k(i) - 1) \leq P_t^k - b_i \quad \text{for } i = 1..N; t = 1..T; k = 1..K. \quad (13)$$

$$z_t^k(i) - z_t^k(i+1) \geq 0 \quad \text{for } i = 1..N; t = 1..T; k = 1..K. \quad (14)$$

$$b_i \leq b_{i+1} \quad \text{for } i = 1..N. \quad (15)$$

$$1 - r_i \leq M\Delta q_i \quad \text{for } i = 1..N. \quad (16)$$

$$\Delta q_i \leq Q^{\max} (1 - r_i) \quad \text{for } i = 1..N. \quad (17)$$

$$r_i \leq r_{i+1} \quad \text{for } i = 1..N. \quad (18)$$

$$q_t^k = \sum_{i=1}^N z_t^k(i) \Delta q_i \quad q_t^k = \sum_{i=1}^N z_t^k(i) \Delta q_i \quad \text{for } i = 1..N; t = 1..T; k = 1..K. \quad (19)$$

Eq. 3, 4, 5, and 7.

Note that M is a large number, and z and r are binary variables that force the bid prices to be in close proximity to market prices. It is expected that a solution can be found for a limited number of price samples using NLP. It is also important to note that the bidding is a daily process, and a solution should be determined each day within a limited time frame.

3.3. Marginal Cost Bidding

In a perfectly competitive market, each player is expected to submit its costs as it will get paid by MCP when selected if it is in a uniform price auction. Such an approach is practical as an offer is selected only if the MCP is larger than the bid price. As an alternative, the power supplier could split the maximum capacity into N blocks of identical sizes, and offer them at prices equal to the marginal costs of producing each block. As the number of blocks is limited in different markets, N can be determined based on the market. Then, for the same problem of Eq. 2, the constraints can be formulated as below.

$$\Delta q_i = \frac{Q^{\max}}{N} \quad \text{for } i = 1..N. \quad (20)$$

$$b_i = a_2 + 2a_3q_i^k \quad \text{for } k = 1..K; t = 1..T; i = 1..N. \quad (21)$$

Eq. 3, 6, and 7.

Such an approach will let the generator run for different levels of market prices as the bidding strategy will have N different prices. The amount of power the generator will supply will be

different at each price level, and the generator will increase the chance of being selected as a unit for dispatch in the market. This approach needs no computational time; hence, it might be preferred by the suppliers who need a reliable but less time-consuming methodology, given that the bidding is a daily process, and it has tight time schedules for bid submission.

3.4. Bid Simulator

In order to evaluate a bidding strategy for given market price scenarios, a simulation methodology can be utilized. The simulation method should include different price samples, and should be able to work for different cost functions. We develop a simulation model called the Bid Simulator that includes market price scenarios and calculates hourly profits according to the market prices to evaluate a bidding curve.

This fundamental model generates a set of electricity market price forecasts, which is required as an input to our proposed offering strategy. If the market price at a particular hour is larger or equal to any given price bid, the supplier would sell power. Otherwise, it would not sell power at that hour. In order to generate market price samples, the simulation methodology uses the Monte Carlo method and mean and variance of the historical prices. The pseudo-code for the simulation is given in Figure 2.

It is obvious that a bidding strategy can return a different profit for different market price scenarios. The methodology provides the expected profit of each bidding strategy over K price samples, and supportive statistical outputs to the decision maker such as statistical outputs, probabilistic distributions and confidence intervals. It is also worth mentioning that a bidding strategy that is found using QP, NLP or marginal cost based bidding can still be evaluated over K price samples. Also, expected profits can be analyzed to make better analyses considering that those solutions are found for limited price samples; and simulation includes K price samples.

4. NUMERICAL STUDIES FOR BIDDING

The proposed models are tested in the PJM market mechanism using PJM market prices. The power producer needs to determine his/her bidding offer which consists of at the most 10 prices (\$/MWh) and corresponding quantity MWh pairs to be submitted

Figure 2: Pseudo code of the simulation model

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0: Determine bid prices and quantities, bi and Δqi
1: Generate K price samples each for 24 hours
2: For each K
3:   For each hour t
4:     For each block i in the bidding curve
5:       If bidding price bi <= market price Ptk
6:         Calculate hourly profit
7:       Else
8:         Hourly profit=0
9:     Next block i
10:  Next hour
11:  Daily Profit = Sum (Hourly profit)
12: Next sample
13: Average Profit = Sum (Daily Profit )/K
    
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to ISO until noon each day. ISO will collect sell bids and buy bids consecutively will run the Security Constrained Economic Dispatch mechanism in which the resources are assigned based on their offer characteristics; and day ahead market price is determined for each hour based on the cumulative supply and demand. The market price data is released regularly paying attention to confidential cost data of each supplier. As the proposed methodologies consider price taker bidding strategies, we have arbitrarily selected 10 price samples from PJM power market. Figure 3 shows the price samples.

Note that PJM market is one of the largest power markets, and the shape of the market prices is affected by habits and work hours. The demand is low at night, but it starts to increase as people go to their daily routines, and it is higher in the evening with different patterns on weekdays, weekends, and on public holidays. We consider a thermal generator whose cost function is $c(q) = 45q + 0.0042q^2$ with a maximum capacity of 300 MW. Such a generator can be considered a price taker, and he has no power to affect with such a capacity the day ahead market price. The supplier is willing to accept the market price, and he needs to submit a bid strategically in an effort to cover different price levels and maximize its expected profit. The supplier might target specific hours in the day-ahead market, and prefer to reach an optimum solution. For such a case, the QP model can be used to solve the problem by setting the time

Table 1: Optimum solution to QP problem

Block	1	2	3	4	5	6
b_i (\$/Mwh)	45.50	45.90	46.10	46.80	47.10	50.20
q_i (Mwh)	59.52	107.14	130.95	214.28	250.00	300.00

Table 2: Optimum solution to NLP problem

Block	1	2	3	4	5	6	7	8	9
b_i (\$/MWh)	45.01	45.54	46.10	46.16	46.39	46.52	46.56	46.87	47.56
q_i (MWh)	51.19	65.47	130.94	138.08	165.46	180.93	211.88	222.59	261.28

Table 3: Bid prices and quantities for marginal cost bidding

Block	1	2	3	4	5	6	7	8	9	10
b_i (\$/MWh)	45.25	45.50	45.75	46.00	46.26	46.51	46.76	47.01	47.26	47.52
q_i (MWh)	30	60	90	120	150	180	210	240	270	300

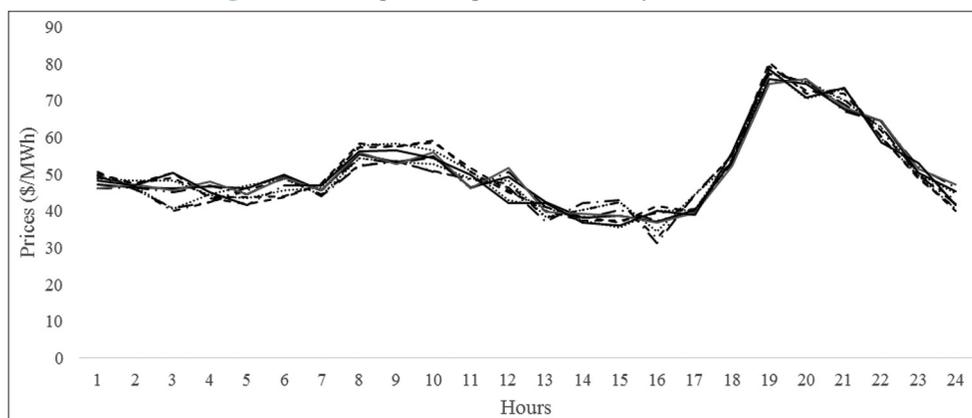
horizon to 10 h as it is equivalent to the number of bidding blocks. The solution for 10 h market prices is given in Table 1. After solving the above model using Cplex, the optimal profit is found to be \$1772.48.

In order to increase the reliability of his bidding strategy, the supplier can include more price samples and can find a bidding strategy. As QP is limited with the number of blocks, the bidding problem under market price uncertainty is solved with NLP. The problem is structured in AMPL and submitted to one of the NEOS Servers MINLP to get a solution. After a number of iterations and computational time, a solution is found for 3 day price samples. Table 2 provides the optimum solution. The objective function of the optimum solution was \$44,779.83. However, it takes about 5 h to solve the problem with 3 price samples and 300 Mwh capacity. If we increase the capacity to 1500 Mwh and try to solve the problem with the same price samples, we could not find an optimal solution after a 24-h run. Results show that it is not likely to solve the problem with more than 3 price samples.

The marginal cost bidding model requires splitting the maximum capacity into equal block sizes. PJM accepts a maximum of ten energy blocks in its daily bidding process, so the maximum capacity can be split into 10 blocks, and the marginal cost of these quantities can be offered to the market. We solve the problem by using the same generator with the price samples given in Figure 2. Table 3 gives the price and quantities for marginal cost bidding.

If a high variability is expected at the market prices, the supplier prefers the marginal cost based bidding model in an effort to

Figure 3: Market price samples from PJM day ahead market



decrease his risk, cover the different price levels, and increase the chance of being selected as a dispatched unit. The marginal cost model is evaluated for both price samples used in QP and NLP. The profit found for 10-h price sample is \$1766.58 where the optimum solution in QP is \$1,772.48. The profit found in 3 day price samples is \$44,750.86 where the optimum solution in NLP is \$44,779.83. Although the profit increases by 0.33% and 0.06% in QP and NLP respectively might seem to be small, such numbers represent huge gains considering the volume of the transactions for each operation day. We also verified the solutions using a bid simulator.

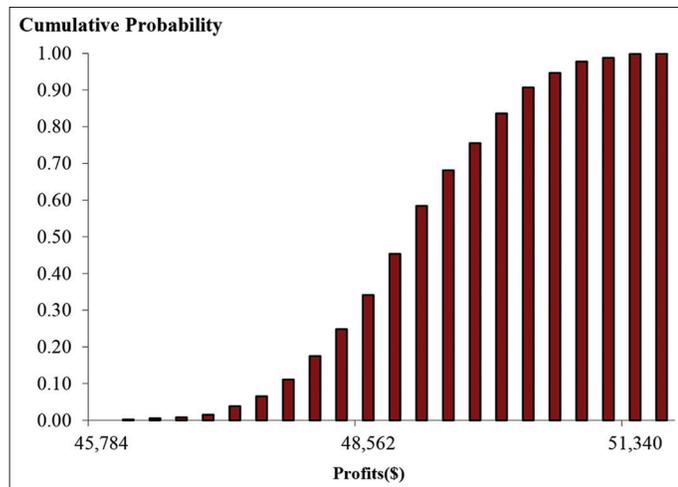
It is also possible to increase the number of price samples and evaluate the effectiveness of each bidding strategy using a bid simulator. The success level of a bidding strategy is related to its effectiveness against different market price scenarios. A bid simulator is designed to include the desired number of market prices from different power markets to increase the reliability of a bidding strategy. The solution to the marginal cost based bidding model is to evaluate in the bid simulator using 10 price samples given in Figure 3. The supportive statistics for the decision making process is given in Table 4.

It is also possible to calculate the mean profit with a defined confidence interval. 5% confidence interval for the mean profit is \$49709.59 and \$49954.39. The distribution of the profits along with the probabilities are provided in an effort to support the decision making process. Figure 4 provides the cumulative distribution function of the profits.

Table 4: Statistics for the expected profits

Parameter	Value
Maximum profit (\$)	53340.23
Minimum profit (\$)	47784.47
Expected profit (\$)	49831.99
Standard deviation	1974.83
Variation	3899952
95% percentile (\$)	53083.52
5% percentile (\$)	47870.75

Figure 4: Market price samples from PJM day ahead market



5. CONCLUSIONS AND POLICY IMPLICATIONS

The bidding process is daily and the suppliers need to make a decision within a limited time frame. The bidding strategy should be carefully determined according to the price taker or price maker nature of the unit. In this paper, the strategic bidding model for price taker units is analysed; and possible solution approaches and their limitations are explained. It is shown that QP is able to find a solution for small problems where the number of price scenarios is limited. NLP can find a solution for more price scenarios, but as the problem gets larger, it gets difficult to find a solution. The solution method should require low computational time as there is a tight schedule each day. As another alternative, it is shown that a generator can turn its marginal cost into bidding blocks and submit them to the market.

The simulation methodology is used to evaluate the bidding offers found in Quadratic Programming, nonlinear programming, and marginal cost bidding, as well as to present the statistical results for each offer. The bid simulator can be used to extend the analysis to increase the number of price samples, and the presented statistical results can be used. Also, a sensitivity analysis can be performed for the decision making process. The presented models and solution approaches, besides filling a gap in the literature, can be used by market players if they do not affect the market price and have imperfect information about market prices. The paper proposes fast and reliable solution methods to the strategic bidding problem that can be adapted by the suppliers, and the proposed simulation methodology has the potential to help decision makers when evaluating a bidding strategy.

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