



## Asymmetric Threshold Cointegration and Nonlinear Adjustment between Oil Prices and Financial Stress

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Received: 10 December 2018

Accepted: 03 March 2019

DOI: <https://doi.org/10.32479/ijeeep.7643>

### ABSTRACT

This paper attempts to estimate the relationship between oil prices and financial stress using weekly data for the period December 31, 1993 to July 15, 2016. The analysis is carried out using the cointegration framework. Both the linear and non-linear models for cointegration and related error correction models are estimated. The paper finds the threshold cointegration model more suitable than the linear cointegration models. It finds evidence of asymmetry in the adjustment process to equilibrium. It also finds that regimes with negative (below the threshold) changes of deviations adjust much faster than regimes with positive (above the threshold) changes of deviations, especially during a crisis period. Also, bi-direction causality is reported between the two variables.

**Keywords:** Threshold Cointegration, Asymmetric Adjustment, Asymmetric Error Correction, Financial Stress, Oil Prices, Financial Crisis

**JEL Classifications:** C22, C32, C58, G11

### 1. INTRODUCTION

In the literature, it is well recognized that oil price shocks have detrimental effect on economic activity in developed and developing countries (Cunado and de Gracia, 2003; Cunado and de Gracia, 2005; Hamilton, 2011), especially for oil-importers. Although the effect of oil shocks on the macro-economy seems to have weakened through time, Kilian (2008) argued that this is partly due to increased demand for industrial output, which offsets the negative impact of an increase in oil price. Besides, Rafiq et al. (2009) argued that the negative impact of an increase in oil price is usually found to be higher than the positive impact of a fall in oil prices. In addition, Balke et al. (2002) claimed that monetary policy alone cannot account for this asymmetry. Transaction costs and financial stress are among the factors that lead to the asymmetric effect.

The financial stress index literature is a rapidly developing one. Existing studies either focus on only constructing a financial stress index for a single country (Illing and Liu, 2006; Hakkio and

Keeton, 2009; Morales and Estrada, 2010; Holló, 2012; Nazlioglu et al., 2015) or both on constructing an index for numerous countries and evaluating the link between financial stress and economic activity to examine how well financial stress index identifies known periods of financial distress (Slingenberg and de Haan, 2011; Cardarelli et al., 2011; Holló et al., 2012; Cevik et al., 2013; Mallick and Sousa, 2013; Chau and Deesomsak, 2014; Islami and Kurz-Kim, 2014). For one country, financial stress indices combine more indicators into one statistic than multi-country stress indices (Kliesen et al., 2012; Vermeulen et al., 2015). Most studies use market data, but some of them use both mixed market and balance sheet data (Holló et al., 2012), while others consider only balance sheet data (Morales and Estrada, 2010).

Different ways are used by authors in order to combine indicators into an aggregate financial stress index. While most studies take the average of standardized variables, others use principal components analysis (Illing and Liu, 2006; Hakkio and Keeton, 2009). More recently, authors used portfolio theory based aggregation schemes

that take into account the correlation structure of stress indicators in order to quantify the level of systemic stress (Holló et al., 2012).

The financial stress index is a relatively new concept, and the literature on impacts of oil prices on macroeconomic and financial variables is a lot broader than the financial stress index literature. The work developed by Chen et al. (2014) is probably the first study in the literature that examines the link between financial stress and oil prices. In their article, Chen et al. (2014) extended the Killian (2009) framework to identify an exogenous shock arising from changes in financial market conditions and examine the consequent macroeconomic impacts of oil price changes. Using the Kansas City financial stress index, global oil production, global real economic activity, and real oil prices, they found that financial stress index shocks trigger a significant negative response in real oil prices.

Furthermore, a second study by Nazlioglu et al. (2015) examined whether there is a volatility transmission between oil prices and financial stress by means of the volatility spillover test. They used west texas intermediate (WTI) crude oil prices and Cleveland financial stress index (CFSI) for the period 1991-2014 and divide the sample into pre-crisis, in-crisis, and post-crisis periods due to the downward trend in oil price in 2008. According to these authors, the dynamic relationship between oil prices and financial stress can exist through two channels: Their impact on economic activity and on investor behavior. Their empirical results indicate that oil prices and financial stress index are dominated by long-run volatility. They also argued that a rise in oil prices depresses economic activity, may put pressure on credit markets, and negatively affect stock markets and the banking system. Besides, they claimed that, in times of high financial stress, economic activity slows down, leading to low energy demand and declining oil prices. In addition, crude oil markets are seen by investors as alternative investment areas to financial markets. With respect to oil price shocks, investors adjust their portfolios, which will have repercussions on financial asset prices. On the other hand, investors will be obliged to change their portfolios due to increased financial stress, which will have an effect on oil markets. Furthermore, Nazlioglu et al. (2015) claimed that financial stress influences economic activity through the bank lending channel via decreasing the amount of available credits and through financial leverage via changes in credit worthiness of borrowing businesses.

Even though the relationship between financial stress and real economic activity or growth is well-studied (Demirguc-Kunt and Levine, 2001; Levine, 2005; Illing and Liu, 2006), the inter-temporal link between oil price and financial stress index is not yet well explored. This study examines whether there is asymmetric relationship between world oil prices and financial stress index. Considering the leading role of the U.S. financial system all over the world, the financial stress index for U.S. is taken as representative of the global financial stress. To the extent of our knowledge, this study is the first to explicitly examine asymmetric threshold cointegration between financial stress and world oil markets by employing the methodology developed by Enders and Siklos (2001).

Most of studies adopt a linear cointegration framework (Engle and Granger, 1987), which assumes a linear long-run relationship among economic variables and a linear adjustment towards the equilibrium. However, the linear structure has been challenged as many economic variables display a nonlinear or asymmetric effect in their long-run relationship and short-term adjustment process (Granger and Lee, 1989; Enders and Granger, 1998). For cross-listings between crude oil prices and financial stress, the rationale of nonlinear modeling is more straightforward. Given the intricacies of the trading environment between these two markets, such as transaction costs, short-sell restrictions and exchange rate risks, arbitragers may only appear when price deviations from the equilibrium are large enough to cover their transaction costs and risk premia, implying an asymmetric adjustment process. There have been some extensions of linear cointegration models to capture this asymmetry. For instance, Balke and Fomby (1997) generalize the cointegration analysis to allow for a threshold effect in the adjustment process. Enders and Granger (1998) and Enders and Siklos (2001) expand the Engle-Granger two-step cointegration test by allowing for the possibility of asymmetric adjustment processes.

In this study, we employ the Enders-Siklos threshold cointegration test to explore the long run asymmetric equilibrium relationship between oil prices and financial stress. The data set includes weekly observations from December 31, 1993 to July 15, 2016, and is divided into three sub-periods due to the downward trend in oil prices in 2008: The pre-oil crisis, the oil crisis, and the post oil crisis (pre-crisis, in-crisis, and post-crisis hereafter) periods.

The remainder of the article is organized as follows. Section 2 describes the data. Section 3 outlines the econometric methodology. Section 4 presents the descriptive statistics and time series properties of data and discusses the empirical results. Section 5 is devoted to concluding remarks.

## 2. DATA SOURCES AND DESCRIPTION

In this study, we use two variables, namely a measure of financial stress and oil prices at weekly frequency. The decision to carry out the analysis at weekly frequency is to better account for the dynamic relationships between oil and financial markets during the 2007-2009 global financial crisis. For world oil prices, we use the WTI spot crude oil prices, obtained from the FRED database of the St. Louis Federal Reserve Bank<sup>1</sup>. Given that our oil prices are weekly, we employ the St. Louis Fed's Financial Stress Index (STLFSI)<sup>2</sup> provided by the Federal Reserve Bank of St. Louis. While there are other financial stress index measures for the US, like the Chicago fed index, the Kansas city fed index (KCFSI), and the CFSI<sup>3</sup>, none of these indexes are available at weekly frequency.

1 <https://fred.stlouisfed.org>.

2 The St. Louis Fed's Financial Stress Index (STLFSI) is based on 18 weekly data series. The actual index is constructed using a principal components analysis, which is a statistical method of extracting factors responsible for the comovements of the 18 variable groups. It is assumed that financial stress is the primary factor influencing this comovement, and by extracting this factor (the first principal component) financial stress index can be created.

3 Note that the CFSI, as a measure of stress in financial markets, has been unavailable since May 9, 2016, due to the discovery of errors that overestimated stress in the real estate and securitization markets.

The STLFSI measures the degree of financial stress in the markets. This index is more comprehensive and overcomes the potential criticisms of focusing solely on one indicator. In combining several indicators, it has a broad coverage as it covers three important areas: (i) Interest rates (such as federal fund rate; 2 year, 10 year, and 30 year treasury; and corporate bond yield); (ii) yield curve (such as 10 year minus 3-month treasury; corporate bond minus 10 year treasury; 3 month TED spread); and (iii) other counterparty risk indicators (such as J.P. Morgan emerging markets bond index, Chicago board options exchange market volatility index, Merrill lynch bond market volatility index). Each of these variables captures some aspect of financial stress. The average value of the index, which begins in late 1993, is designed to be zero. Thus, zero is viewed as representing normal financial market conditions. Values below zero suggest below-average financial market stress, while values above zero suggest above-average financial market stress.

Note that an increase in financial stress will be associated with higher funding costs and greater economic uncertainty, resulting in declining real economic activity. Moreover, an increased financial stress will render financial investors more risk averse, which will discourage investment in asset markets, resulting in falling asset prices, including oil prices (Hakkio and Keeton, 2009; Davig and Hakkio, 2010).

The data set includes weekly observations from December 31, 1993 to July 15, 2016 and it is divided into three sub-periods: The pre-crisis period from December 31, 1993 to July 27, 2007, the crisis period from August 3, 2007 to March 27, 2009, and the post-crisis period from April 3, 2009 to July 15, 2016. Even though the WTI starts earlier, the starting date of the sample is constrained by availability of STLFSI, and all available data since the start of this study is included.

The recent global financial crisis has some unique features, such as the length, breadth, and crisis sources. Numerous studies use major economic and financial events in order to determine the crisis length and source ad hoc (Baur, 2012; Dimitriou and Kenourgios, 2013; Dimitriou et al., 2013; Mighri and Mansouri, 2014). Besides, the choice of the sub-periods is based on the downward trend in oil prices within the crisis date (Mollick and Assefa, 2013; Turhan et al., 2013). These studies suggested that when oil prices are used, separate analyses are necessary before, at and after the crisis period.

In this study, the length of the global financial crisis and its phases are specified following an economic approach. We define a relatively long crisis period based on all major international financial and economic news events representing the global financial crisis. We use the official timelines provided by Federal Reserve Board of St. Louis (2009) and the BIS (2009), among others, in order to choose the crisis period. According to these studies, the timeline of the global financial crisis is separated in four phases. Phase 1 described as “initial financial turmoil spans from 1 August 2007 to 15 September 2008. Phase 2 is defined as “sharp financial market deterioration” and spans from 16 September 2008 to 31 December 2008. Phase 3 described as

“macroeconomic deterioration” spans from January 1, 2009 to March 31, 2009. Phase 4 described as a phase of “stabilization and tentative signs of recovery” (post-crisis period), including a financial market rally, spans from 1 April 2009 to the end of the sample period. For that reason, the crisis can be defined from August 2007 to March 2009 covering the first three phases. In the light of the literature, we therefore question the impact of the recent global financial crisis on the financial stress and oil price link and thus the data is divided into three sub-periods.

### 3. ECONOMETRIC METHODOLOGY

Cointegration has been widely used to investigate relationship among price variables. The two major cointegration methods are Johansen and Engle-Granger two-step approaches. Both of them assume symmetric relationship between variables. In recent years, threshold cointegration has been increasingly used in price transmission studies. Balke and Fomby (1997) proposed a two-step approach for examining threshold cointegration on the basis of the approach developed by Engle and Granger (1987). Enders and Granger (1998) and Enders and Siklos (2001) further generalize the standard Dickey-Fuller test by allowing for the possibility of asymmetric movements in time-series data. This makes it possible to test for cointegration without maintaining the assumption of a symmetric adjustment to a long-term equilibrium. Thereafter, the method has been widely applied to analyze asymmetric price transmission.

In this study, linear cointegration, threshold cointegration, and asymmetric error correction models are employed to examine the oil price and financial stress dynamics. These models will be able to assess asymmetric price dynamics in both the long term and short term.

#### 3.1. Linear Cointegration Analysis

In this study, the focus variables are weekly as well as monthly price series of crude oil and financial stress in the United States. As usual, their stochastic properties of non-stationarity and order of integration can be evaluated using the Augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979), and Phillips-Perron (Phillips and Perron, 1988) unit root tests. If both the price series appear to have a unit root, then it is appropriate to conduct cointegration analysis to assess their interaction. Econometric literature proposes different methodological alternatives to empirically analyze the long-run relationships and dynamics interactions between two or more time-series variables. Two cointegration methods widely used are the full information maximum likelihood-based Johansen approach and Engle-Granger two-step approach (Engle and Granger, 1987; Enders, 2004). The Johansen approach is a multivariate generalization of the Dickey-Fuller test (Johansen, 1988; Johansen and Juselius, 1990). It concentrates on the relationship between the rank of a matrix and its characteristic roots in a vector autoregression.

The Johansen approach starts with a vector autoregressive model and then reformulates it into a vector error correction model as follows:

$$V_t = \pi_1 V_{t-1} + \dots + \pi_k V_{t-k} + \varepsilon_t \quad (1)$$

$$\Delta V_t = \sum_{i=1}^{K-1} \Gamma_i \Delta V_{t-i} + \Pi V_{t-K} + \varepsilon_t \quad (2)$$

Where  $V_t$  is a vector of the price at date (week)  $t$  for crude oil ( $y_t$ ) and financial stress ( $x_t$ ),  $K$  is the number of lags, and  $\varepsilon_t$  is the error term. The relationship among the coefficients for the two equations is given as:

$$\Gamma_i = -I + \sum_{j=1}^p \pi_j \quad (3)$$

$$\Pi = -I + \sum_{h=1}^K \pi_h \quad (4)$$

Where  $I$  is an identity matrix. Two types of tests, i.e., the trace and maximum eigenvalue statistics, can be used to detect the number of cointegrating vectors,  $r$ , among the variables in  $V_t$ .

The Engle-Granger two-stage approach focuses on the time series property of the residuals from the long-term equilibrium relationship (Engle and Granger, 1987). The first step of the analysis consists in determining a break point into the relationship that defines the long run relationship between the crude oil prices and financial stress index:

$$y_t = \zeta_0 + \zeta_1 x_t + \varepsilon_t \quad (5)$$

$$\Delta \varepsilon_t = \rho \varepsilon_{t-1} + \sum_{i=1}^p \varphi_i \Delta \varepsilon_{t-i} + z_t \quad (6)$$

Where  $y_t$  and  $x_t$  denote the oil prices and financial stress, respectively,  $\zeta_0$ ,  $\zeta_1$ ,  $\rho$  and  $\varphi_i$  are parameters to be estimated,  $\varepsilon_t$  is the disturbance term, which should be stationary if any long-run relationship exists between the two integrated price series, indicates the first difference,  $\hat{\varepsilon}_t$  is the estimated residuals,  $\rho$  measures the speed of convergence of the system,  $z_t$  is a white noise disturbance term, and  $p$  denotes the number of lags. The parameter  $\zeta_1$  indicates the long-run elasticity of price transmission and gives the magnitude of adjustment of the crude oil price to variations of the financial stress index. If  $\zeta_1 < 1$ , changes in the financial stress index are not fully passed onto the crude oil price.

In the first stage of estimating the long-term relationship among the variables  $y_t$  and  $x_t$ , the financial stress is chosen to be placed on the right side and assumed to be the driving force. In the second stage, the estimated residuals  $\hat{\varepsilon}_t$  are used to conduct a unit root test (Engle and Granger, 1987). The number of lags is chosen so there is no serial correlation in the regression residuals. It can be selected using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), or Ljung-Box  $Q$  test. If the null hypothesis of  $\rho=0$  is rejected, then the residual series from the long-term equilibrium is stationary and the focal variables of  $y_t$  and  $x_t$  are cointegrated. Rejecting the null hypothesis of no cointegration  $\rho=0$  in favor of the alternative hypothesis  $-2 < \rho < 0$  implies that the  $\{\varepsilon_t\}$  sequence is stationary with mean zero. Any deviations from the long-run value of the disturbance term  $\varepsilon_t$  are ultimately eliminated. Convergence is assured if  $-2 < \rho < 0$ . As such, Eq. (5) is an attractor such that  $\varepsilon_t$  can be written as an

error correction model. The change in  $\varepsilon_t$  equals  $\rho$  multiplied by  $\varepsilon_{t-1}$  regardless of whether  $\varepsilon_{t-1} \geq 0$  or  $\varepsilon_{t-1} < 0$ .

### 3.2. Cointegration Analysis with Structural Breaks

Residual-based cointegration tests (Engle and Granger, 1987) assume that cointegrating vectors are constant over time. However, if there is a regime shift in the series, there will be a shift in the cointegrating vector as well. In such circumstances, these standard tests could lead to incorrect inferences about the long-run relationship of the price series. Furthermore, Phillips (1986) shows that if a structural break exists in the data, but is omitted from the cointegration relationship, this could lead to spurious rejections when the null of no cointegration is wrongly rejected. For the Engle and Granger (1987) test, such spurious rejections tend to occur for breaks that are located either too early in the sample or when the magnitude of the break increases. Thus, the power of the Engle and Granger (1987) test to find cointegration is severely affected by the presence of breaks in the level or the trend function in the cointegration relationship.

Gregory and Hansen (1996a; 1996b) addressed this issue and proposed a residual-based cointegration test that allows for the possibility of regime shifts either in the intercept or the entire vector of coefficients. Gregory and Hansen (1996a; 1996b) analyzed four models and then tested the null hypothesis of no cointegration. Model 1 (see Eq.5) is the standard cointegration model where no changes in the intercept or a trend function are allowed under the null hypothesis. The other three models include shifts in either the intercept (Level shift model  $C$ ) or trend (Level shift model with trend  $C/T$ ) or shifts in the intercept and slope vector of coefficients (Regime shift model  $C/S$ ). Model  $C/S$  is unique in the sense it allows the long-run equilibrium relationship to rotate as well as shift in parallel fashion. The break point in any model is determined endogenously within the data series.

Level shift model  $C$  can be expressed as follows:

$$y_t = \xi_0' + \xi_0'' \phi_{t_0} + \xi_1' x_t + \varepsilon_t \quad (7)$$

In this parameterization,  $\xi_0'$  represents the intercept before the shift, and  $\xi_0''$  denotes the change in the intercept at the time of the shift.

Level shift model with trend  $C/T$  can be represented by:

$$y_t = \xi_0' + \xi_0'' \phi_{t_0} + \beta t + \xi_1' x_t + \nu_t \quad (8)$$

Regime shift model  $C/S$  is given as:

$$y_t = \xi_0' + \xi_0'' \phi_{t_0} + \xi_1' x_t + \xi_1'' x_t \phi_{t_0} + \kappa_t \quad (9)$$

In this case,  $\xi_0'$  and  $\xi_0''$  are as in the level shift model  $C$ .  $\xi_1'$  denotes the cointegrating slope coefficients before the regime shift, and  $\xi_1''$  denotes the change in the slope coefficients.

A time trend into the regime shift model ( $C/S/T$ ) could be also introduced:

$$y_t = \xi_0 + \xi_0 \phi_{t_0} + \beta t + \xi_1 x_t + \xi_1 x_t \phi_{t_0} + \eta_t \quad (10)$$

In these four models above, the structural break is modeled by the introduction of a dummy variable  $\phi_{t_0}$ , which takes values (0,1) depending on the nature of the structural break.

$$\phi_{t_0} = \begin{cases} 1 & \text{if } t > t_0 \\ 0 & \text{if } t \leq t_0 \end{cases} \quad (11)$$

Where  $t_0$  is the unknown parameter denoting the timing of the change point.

In all four models postulated, the null hypothesis of no cointegration can be tested by examining whether the residuals of the ordinary least-squares (OLS) regression applied to Eqs. (5) and (7)-(10), respectively are stationary processes.

The procedure for computing the test statistic for each possible regime shift  $t_0 \in T$  involves four steps. In essence, it involves the search for the smallest value of either the modified Phillips-Perron (PP) ( $Z_\xi^*$  and  $Z_t^*$ ) or ADF(ADF\*) test statistic across all possible break points:

$$Z_\xi^* = \inf_{t_0 \in T} Z_\xi(t_0) \quad (12)$$

$$Z_t^* = \inf_{t_0 \in T} Z_t(t_0) \quad (13)$$

$$ADF^* = \inf_{t_0 \in T} ADF(t_0) \quad (14)$$

### 3.3. Threshold Cointegration Analysis

The implicit assumption of linear and symmetric adjustment (Engle and Granger, 1987) is problematic. Enders and Siklos (2001) proposed a two-regime threshold cointegration approach to entail asymmetric adjustment in cointegration analysis. They argued that the Engle-Granger cointegration test is likely to lead to misspecification errors when the adjustment of the error correction term is asymmetric. They remedy this error by expanding the Engle-Granger two-step cointegration test to incorporate an asymmetric error correction term. In the next step, we determine whether or not the disturbance term  $\varepsilon_t$  is stationary by considering an asymmetric test methodology in the form of Threshold Autoregressive (TAR) cointegration model as proposed by Enders and Granger (1998) and Enders and Siklos (2001). The alternative model modifies Eq. (6) such that:

$$\Delta \varepsilon_t = I_t \rho_1 (\varepsilon_{t-1} - \tau) + (1 - I_t) \rho_2 (\varepsilon_{t-1} - \tau) + \mu_t \quad (15)$$

Where  $\rho_1, \rho_2$  are coefficients,  $\tau$  is the value of the threshold,  $\mu_t$  is a white-noise disturbance and  $I_t$  is the Heaviside indicator such that:

$$I_t = \begin{cases} 1 & \text{if } \varepsilon_{t-1} \geq \tau \\ 0 & \text{if } \varepsilon_{t-1} < \tau \end{cases} \quad (16)$$

In order for  $\{\varepsilon_t\}$  to be stationary, a necessary condition is  $-2 < (\rho_1, \rho_2) < 0$ . If the variance of  $\mu_t$  is sufficiently large, it is also possible for one value of  $\rho_j$  to be in the range of  $-2$  and  $0$  and for the other value to equal zero. Although there is no convergence in the regime with the unit-root (i.e., the regime in which  $\rho_j = 0$ ), large realizations of  $\mu_t$  will switch the system into the convergent regime.

In both cases, under the null assumption of no cointegration between the variables, the  $t$ -statistic for the null hypothesis  $\rho_1 = \rho_2 = 0$  has a nonstandard distribution. Rejecting this assumption means that Eq. (15) is an attractor such that the equilibrium value of the  $\{\varepsilon_t\}$  is  $\tau$ . The adjustment process is  $(\rho_1 \varepsilon_{t-1} - \tau)$  if the lagged value of  $\varepsilon_t$  is above its long-run equilibrium value, while if the lagged value of  $\varepsilon_t$  is below its long-run equilibrium value, the adjustment is  $\rho_2 (\varepsilon_{t-1} - \tau)$ . If  $-1 < |\rho_1| < |\rho_2| < 0$ , negative discrepancies will be more persistent than positive discrepancies. Moreover, Tong (1983) showed that the OLS estimates of  $\rho_1$  and  $\rho_2$  have an asymptotic multivariate normal distribution if the sequence  $\{\varepsilon_t\}$  is stationary. Therefore, if the null assumption  $\rho_1 = \rho_2 = 0$  is rejected, it is possible to test for symmetric adjustment (i.e.,  $\rho_1 = \rho_2$ ) using a standard F-test. Rejecting both the null assumptions  $\rho_1 = \rho_2 = 0$  and  $\rho_1 = \rho_2$  indicates the existence of threshold cointegration and asymmetric adjustment.

Since the exact nature of the nonlinearity may not be known, Enders and Siklos (2001) consider another kind of asymmetric cointegration test methodology that allows the adjustment to be contingent on the change in  $\varepsilon_{t-1}$  (i.e.,  $\Delta \varepsilon_{t-1}$ ) instead of the level of  $\varepsilon_{t-1}$ . In this case, the Heaviside indicator of Eq. (16) becomes.

$$I_t = \begin{cases} 1 & \text{if } \Delta \varepsilon_{t-1} \geq \tau \\ 0 & \text{if } \Delta \varepsilon_{t-1} < \tau \end{cases} \quad (17)$$

This specification is especially relevant when the adjustment is such that the series exhibits more “momentum” in one direction than in the other (Thompson, 2006; Kuo and Enders, 2004; Enders and Siklos, 2001; Enders and Granger, 1998). That is, the speed of adjustment depends on whether  $\varepsilon_t$  is increasing (i.e., widening) or decreasing (i.e., narrowing). According to Thompson (2006), among others, if  $|\rho_1| < |\rho_2|$ , then increase in  $\varepsilon_t$  tend to persist, whereas decreases revert back to the threshold quickly. The resulting model is called momentum-threshold autoregressive (M-TAR) cointegration model. The TAR model captures asymmetrically deep movements if, for instance, positive deviations are more prolonged than negative deviations. The M-TAR model allows the autoregressive decay to depend on  $\Delta \varepsilon_{t-1}$ . As such, the M-TAR specification can capture asymmetrically “sharp” movements in  $\{\varepsilon_t\}$  sequence (Caner and Hansen, 2001).

In both the TAR and M-TAR cointegration processes, the null assumption of  $\rho_1 = \rho_2 = 0$  could be tested, while the null hypothesis of symmetric adjustment may be tested by the restriction,  $\rho_1 = \rho_2$ . Generally, there is no presumption to whether to use TAR or M-TAR specifications. Thus, it is recommended to select the adjustment mechanism by a model selection criterion such as AIC or BIC. Furthermore, if the errors in Eq. (15) are serially correlated, it is possible to use the augmented form of the test:

$$\Delta \varepsilon_t = I_t \rho_1 (\varepsilon_{t-1} - \tau) + (1 - I_t) \rho_2 (\varepsilon_{t-1} - \tau) + \sum_{i=1}^p \varphi_i \Delta \varepsilon_{t-i} + v_t \quad (18)$$

To use the tests, we first regress  $\varepsilon_t$  on a constant and call the residuals,  $\{\hat{\varepsilon}_t\}$  which are the estimates of  $(\varepsilon_{t-1} - \tau)$ . In a second step, we set the indicator according to Eq. (16) or Eq. (17) and estimate the following regression:

$$\Delta \hat{\epsilon}_t = I_t \rho_1 (\hat{\epsilon}_{t-1} - \tau) + (1 - I_t) \rho_2 (\hat{\epsilon}_{t-1} - \tau) + \sum_{i=1}^p \varphi_i \Delta \hat{\epsilon}_{t-i} + v_t \tag{19}$$

The number of lags  $p$  is specified to account for serially correlated residuals and it can be selected using AIC, BIC, or Ljung-Box  $Q$  test. In several applications, there is no reason to expect the threshold to correspond with the attractor (i.e.,  $\tau=0$ ). In such circumstances, it is necessary to estimate the value of along with the values of  $\rho_1$  and  $\rho_2$ . A consistent estimate of the threshold  $t$  can be obtained by adopting the methodology of Chan (1993). A super consistent estimate of the threshold value can be attained with several steps. First, the process involves sorting in ascending order the threshold variable, i.e.,  $\hat{\epsilon}_{t-1}$  for the TAR model or the  $\Delta \hat{\epsilon}_{t-1}$  for the M-TAR model. Second, the potential threshold values are determined. If the threshold value is to be meaningful, the threshold variable must actually cross the threshold value (Enders, 2004). Thus, the threshold value  $\tau$  should lie between the maximum and minimum values of the threshold variable.

In practice, the highest and lowest 15% of the values were removed from the search to ensure an adequate number of observations on each side. The middle 70% values of the sorted threshold variable are used as potential threshold values. Third, the TAR or M-TAR model is estimated with each potential threshold value. The sum of squared errors (SSE) for each trial can be calculated and the relationship between the SSE and the threshold value can be examined. Finally, the threshold value yielding the lowest SSE is deemed to be the consistent estimate of the threshold.

Given these considerations, a total of four models are used in this study. They are TAR- Eq. (16) with  $\tau=0$ ; consistent TAR-Eq. (16) with  $\tau$  estimated; MTAR- Eq. (17) with  $\tau=0$ ; and consistent MTAR- Eq. (17) with  $\tau$  estimated. Since there is generally no presumption on which specification is used, it is recommended to choose the appropriate adjustment mechanism via model selection criteria of AIC and BIC (Enders and Siklos, 2001). A model with the lowest AIC and BIC will be used for further analysis.

Insights into the asymmetric adjustments in the context of a long term cointegration relationship can be obtained with two tests. First, an F-test is used to examine the null assumption of no cointegration ( $H_0: \rho_1 = \rho_2 = 0$ )<sup>4</sup> against the alternative of cointegration with either TAR or M-TAR threshold adjustment. Let  $\Phi$  and  $\Phi^*$  denote the F-statistics for testing the null assumption of  $\rho_1 = \rho_2 = 0$  under the TAR and the M-TAR specifications, respectively. The distributions of  $\Phi$  and  $\Phi^*$  are determined by the form of the attractor. The second one is a standard F-test to assess the null assumption of symmetric adjustment in the long-term equilibrium ( $H_0: \rho_1 = \rho_2$ ). Rejection of the null hypothesis indicates the existence of an asymmetric adjustment process.

4 The null hypothesis of non stationarity is rejected if the sample value of F-test statistic exceeds the Enders-Granger critical value. The critical values of the -statistics for the null hypothesis  $\rho_1 = \rho_2 = 0$  using the TAR and M-TAR specifications are reported in the first and second panels of Table 1 in Kuo and Enders (2004).

### 3.4. Asymmetric Error Correction Model with Threshold Cointegration

According to Engle and Granger (1987), if all considered variables are cointegrated, then there will be a corresponding error correction model (ECM). The finding could be extended to threshold cointegration. This means that, if  $y_t$  and  $x_t$  are threshold cointegrated, then the ECM could be constructed as follows:

$$\Delta x_t = \theta_x + \delta_x^+ Z_{t-1}^+ + \delta_x^- Z_{t-1}^- + \sum_{j=1}^p \alpha_{xj} \Delta x_{t-j} + \sum_{j=1}^p \beta_{xj} \Delta y_{t-j} + v_{x,t} \tag{20}$$

And

$$\Delta y_t = \theta_y + \delta_y^+ Z_{t-1}^+ + \delta_y^- Z_{t-1}^- + \sum_{j=1}^p \alpha_{yj} \Delta x_{t-j} + \sum_{j=1}^p \beta_{yj} \Delta y_{t-j} + v_{y,t} \tag{21}$$

Where  $Z_{t-1}^+ = I_t \hat{\epsilon}_{t-1}$  and  $Z_{t-1}^- = (1 - I_t) \hat{\epsilon}_{t-1}$ ; the parameters  $\delta^+$  and  $\delta^-$  represent the adjustment speed of the coefficients of different sized deviations;  $\theta$  is a constant;  $\alpha_j$  and  $\beta_j$  are the coefficients of the lagged difference terms;  $p$  is the number of lags and  $v_t$  is a white noise. The subscripts  $x$  and  $y$  are used in order to differentiate between the coefficients of variables  $x_t$  and  $y_t$ , respectively.  $t$  denotes time, and  $j$  represents lags.

The equilibrium correction specification (ECM) of Engle and Granger (1987) assumes that the adjustment process due to disequilibrium among the variables is symmetric. In order to incorporate asymmetries, two extensions on the ECM model have been made. Error correction terms and first differences on the variables are decomposed into positive and negative values, as proposed by Granger and Lee (1989). The second extension adds the threshold cointegration mechanism to the Granger and Lee (1989) approach. The resulting asymmetric error correction model with threshold cointegration has the following form:

$$\Delta x_t = \theta_x + \delta_x^+ Z_{t-1}^+ + \delta_x^- Z_{t-1}^- + \sum_{j=1}^p \alpha_{xj}^+ \Delta x_{t-j}^+ + \sum_{j=1}^p \alpha_{xj}^- \Delta x_{t-j}^- + \sum_{j=1}^p \beta_{xj}^+ \Delta y_{t-j}^+ + \sum_{j=1}^p \beta_{xj}^- \Delta y_{t-j}^- + v_{x,t} \tag{22}$$

And

$$\Delta y_t = \theta_y + \delta_y^+ Z_{t-1}^+ + \delta_y^- Z_{t-1}^- + \sum_{j=1}^p \alpha_{yj}^+ \Delta x_{t-j}^+ + \sum_{j=1}^p \alpha_{yj}^- \Delta x_{t-j}^- + \sum_{j=1}^p \beta_{yj}^+ \Delta y_{t-j}^+ + \sum_{j=1}^p \beta_{yj}^- \Delta y_{t-j}^- + v_{y,t} \tag{23}$$

The Heaviside indicator function is constructed from Eq. (16) or Eq. (18). The superscripts “+” and “-” indicate that the variables are split into positive and negative components. The first differences are defined as follows:

$$\Delta x_{t-j}^+ = \max \left\{ \Delta x_{t-j}, 0 \right\} = \begin{cases} x_{t-j} - x_{t-j-1}, & x_{t-j} \geq x_{t-j-1} \\ 0, & x_{t-j} < x_{t-j-1} \end{cases}$$

$$\Delta x_{t-j}^- = \min \left\{ \Delta x_{t-j}, 0 \right\} = \begin{cases} x_{t-j} - x_{t-j-1}, & x_{t-j} < x_{t-j-1} \\ 0, & x_{t-j} \geq x_{t-j-1} \end{cases}$$

$$\Delta y_{t-j}^+ = \max \left\{ \Delta y_{t-j}, 0 \right\} = \begin{cases} y_{t-j} - y_{t-j-1}, & y_{t-j} \geq y_{t-j-1} \\ 0, & y_{t-j} < y_{t-j-1} \end{cases}$$

$$\Delta y_{t-j}^- = \min \left\{ \Delta y_{t-j}, 0 \right\} = \begin{cases} y_{t-j} - y_{t-j-1}, & y_{t-j} < y_{t-j-1} \\ 0, & y_{t-j} \geq y_{t-j-1} \end{cases}$$

The lag  $p$  is specified to account serially correlated residuals and is selected using AIC statistic and Ljung-Box  $Q$  test. The above specifications are able to distinguish between long-run and short-run adjustments of  $x_t$  and  $y_t$ . The long-run adjustment is determined by the parameters  $\delta^+$  and  $\delta^-$ , whereas, the short-run adjustment is governed by the parameters  $\alpha_j^+, \alpha_j^-, \beta_j^+$  and  $\beta_j^-$  for  $j=1, \dots, p$ . If  $\delta_x^+ \neq \delta_x^-$  and  $\delta_y^+ \neq \delta_y^-$ , then both  $x_t$  and  $y_t$  exhibit asymmetry in long-run adjustment. If either  $\alpha_{xj}^+ \neq \alpha_{xj}^-$  or both,  $x_t$  displays asymmetry in short-run adjustment. Besides, if either  $\alpha_{yj}^+ \neq \alpha_{yj}^-$  or  $\beta_{yj}^+ \neq \beta_{yj}^-$  or both,  $y_t$  displays asymmetry in short-run adjustment.

In this paper, four types of single or joint null hypotheses and F-tests are examined (Meyer and Von Cramon-Taubadel, 2004; Frey and Manera, 2007; Sun, 2011; Chen and Zhu, 2015; Mighri and Mansouri, 2016). The first type is the Granger causality test to examine the lead-lag relationship between  $x_t$  and  $y_t$ . The null hypothesis that  $x_t$  does not lead  $y_t$  can be tested by restricting  $H_{01} : \alpha_{yj}^+ = \alpha_{yj}^- = 0$  for all lags  $j$  simultaneously and then employing an  $F$ -test. Similarly, the null hypothesis that  $y_t$  does not lead  $x_t$  can be tested by restricting  $H_{02} : \beta_{xj}^+ = \beta_{xj}^- = 0$  for all lags  $j$  simultaneously and then employing an  $F$ -test. In our empirical analysis, we expect to see one of the following: If one variable Granger-causes the other, then the former variable leads the latter; if there is no causal relationship between the two variables, then there is no obvious connection between the two variables; or, if the two variables mutually Granger-cause each other, then the two variables are closely linked to each other. The second type of hypothesis is concerned with the distributed lag asymmetric effect

on its own variable; that is,  $H_{03} : \alpha_{xj}^+ = \alpha_{xj}^- = 0$  and  $H_{04} : \beta_{yj}^+ = \beta_{yj}^- = 0$ . The third type of the null hypothesis is the cumulative symmetric effect which can be expressed as  $H_{05} : \sum_{j=1}^p \alpha_{xj}^+ = \sum_{j=1}^p \alpha_{xj}^-$  for  $x_t$  and  $H_{06} : \sum_{j=1}^p \beta_{yj}^+ = \sum_{j=1}^p \beta_{yj}^-$ , for  $y_t$ . Finally, the equilibrium adjustment path asymmetry can be examined with the null hypotheses of  $H_{07} : \delta^+ = \delta^-$  for each equation estimated (i.e.,  $\delta_x^+ = \delta_x^-$  for  $x_t$  and  $\delta_y^+ = \delta_y^-$  for  $y_t$ ) to examine whether it is possible to get back to equilibrium after a shock, and if it is the case, how long it will take.

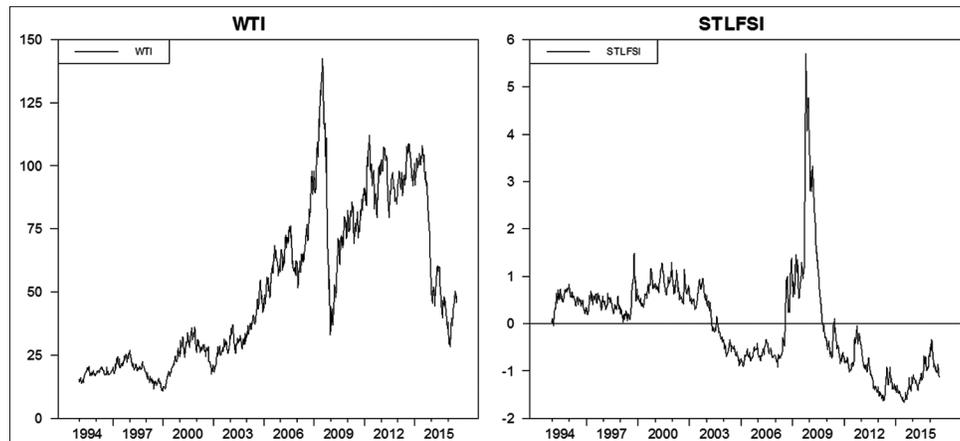
## 4. EMPIRICAL RESULTS

### 4.1. Descriptive Statistics and Unit Root Test

Figure 1 displays the time series plots for the oil prices and St. Louis Fed Financial Stress Index (STLFISI). Three observations can be made (i) Oil prices and STLFISI have an evident comovement in general, which reveals a high possibility of cointegration between these two series. (ii) Although oil prices and STLFISI move together most of the time during our sample period, they also display divergent movement indicating possible nonlinear cointegration. (iii) The two series tend to move more closely during and after the crisis relative to the pre-crisis period. It seems that the link between STLFISI and world oil prices have changed through time, which motivated us to concentrate on the sub-sample analysis.

Table 1 reports summary statistics of oil prices and STLFISI for different samples in order to examine to what extent the descriptive statistics of the oil prices and the STLFISI differ across these sub-periods. The highest mean and standard deviation are observed for oil prices during all the sub-periods. As a simple measurement for volatility, the standard deviations of oil prices are higher in the crisis period compared to those of the pre-crisis and post-crisis periods. Skewness is a simple measure of asymmetry and kurtosis is a measurement for peaked or flatted distribution relative to a Gaussian distribution. We observe that oil price has negative skewness and is left tailed in both the crisis and post-crisis periods; although it has positive skewness and hence right tailed in the pre-crisis period. This stylized fact is

Figure 1: Dynamics of weekly oil prices and St. Louis Fed Financial Stress Index



**Table 1: Descriptive statistics**

Variable	Whole sample		Pre-crisis		In-crisis		Post-crisis	
	WTI	STLFSI	WTI	STLFSI	WTI	STLFSI	WTI	STLFSI
Mean	51.139	0.000	31.351	0.215	87.666	1.798	79.623	-0.811
Median	42.520	0.091	26.340	0.418	91.180	1.099	85.660	-0.934
Maximum	142.520	5.701	75.630	1.470	142.520	5.701	112.300	2.640
Minimum	11.000	-1.657	11.000	-0.906	32.980	-0.077	28.140	-1.657
SD	31.306	1.000	16.670	0.570	29.969	1.483	21.507	0.712
Skewness	0.557	1.299	1.121	-0.479	-0.178	1.039	-0.671	2.124
Kurtosis	2.040	7.514	3.056	1.980	2.059	2.802	2.265	9.015
Jarque-bera	106.004*** (0.000)	1330.457*** (0.000)	148.664*** (0.000)	57.889*** (0.000)	3.670 (0.160)	15.800*** (0.000)	37.155*** (0.000)	860.726*** (0.000)
Observations	1177	1177	709	709	87	87	381	381
Correlation	-0.417	-	-0.758	-	-0.619	-	-0.309	-

\*\*\*, \*\*, and \* indicate respectively statistical significance at the 1%, 5%, and 10% levels. WTI: West texas intermediate, STLFSI: St. Louis fed's financial stress index

also supported by the Jarque-Bera test statistic which rejects the null assumption of normality.

The STLFSI has its positive and the greatest mean in the in-crisis period, which means that there is a significant or higher stress. It has positive mean in the pre-crisis and negative mean in the post-crisis. As illustrated in Figure 1, these features provide that the pre-crisis period is characterized as a normal stress period while moderate stress levels are observed in the post-crisis period. The STLFSI has smallest standard deviation during the crisis-period compared to the pre-and post-crisis periods. The skewness and kurtosis measures indicate deviation from normality. Skewness shows that STLFSI is right tailed in the crisis and post-crisis periods; it has left tail in the pre-crisis period. Kurtosis indicates that financial stress is less peaked during the pre-crisis and in-crisis periods compared to the post-crisis period. Moreover, the Jarque-Bera statistic shows non-normal behavior of financial stress.

The different data characteristics apparent in the summary statistics for different considered periods lead to the question of whether the correlations between oil prices and STLFSI vary across these sub-periods as well. At first glance, correlation appears stronger in the pre-crisis and in-crisis periods than that in the post-crisis period. However, the positive correlation in the in-crisis period turns back to negative following the global financial crisis. This means that as financial stress goes from significant level to moderate level, oil prices go up. This reversed relationship becomes more apparent after 2008 according to Figure 1.

The non-stationarity properties of the oil prices and financial stress indices are investigated by applying the ADF (Dickey and Fuller, 1979), PP (Phillips and Perron, 1988), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (Kwiatkowski et al., 1992) unit root tests. For both ADF and PP tests, we consider three different cases: the case without constant and trend, the case with constant, as well as the case with both constant and trend.

The results for the ADF and PP unit root tests are summarized in both Tables 2 and 3. These tests indicate that oil prices are characterized by a unit root process, implying that the shocks are permanent and not corrected over time. For the STLFSI, even though both the unit root tests support evidence on stationary process in the post-crisis period, this mean reverting process is not supported in the full-sample, pre-crisis and in-crisis periods.

In addition, the KPSS test also reveals that both WTI and STLFSI series fail to reject the alternative assumption of a unit root at the 1% level of significance or better. Conversely, these series accept the null hypothesis of a stationary at 1% level of significance or better when tested for a unit root in first differences. Therefore, we conclude that both oil prices and STLFSI are integrated processes of order one, or unit root processes.

## 4.2. Results of the Linear Cointegration Analysis

To conduct the linear cointegration analyses, we use both the Johansen and Engle-Granger approach. The application of the Johansen approach requires the determination of a lag length for the model, which is based on the lowest AIC and BIC. Without prior information, three model specifications with trend, constant, or no intercept are entailed (Table 4). For instance, with only a trend included, the Johansen maximum eigenvalue statistic ( $\lambda_{max}$ ) is 21.152 for the null hypothesis of no cointegrating vector between the prices of WTI and STLFSI. These are significant at the 5% level, indicating that the null hypothesis is rejected. However, for the null hypothesis of one cointegrating vector, the  $\lambda_{max}$  statistic decreases to 5.472, which is not statistically significant at all. Therefore, the maximum eigenvalue statistic conclude that there is one cointegrating vector. Similarly, the Johansen trace statistic ( $\lambda_{trace}$ ) also supports the conclusion that WTI and STLFSI are cointegrated.

The Engle-Granger cointegration test is implemented through two steps. In the first step, the long-term relationship between the price series is estimated, as specified in Eq. (17). The estimates for the coefficients on the financial stress index (i.e.,  $\xi_j$ ) are highly statistically significant in all cases. In the second step, the residual series is used to conduct a unit root test with the specification in Eq. (18). As reported in Table 5, the sufficiently optimal lag, used for addressing the problem of serial correlation, is chosen based on the AIC and Ljung-Box  $Q$  statistics. Besides, the statistics from the unit root test (i.e.,  $\rho$ ) are statistically significant. The results of the Engle-Granger cointegration test provide evidence for the alternative hypothesis of linear cointegration in all cases except for the post crisis period.

## 4.3. Results of the Cointegration Analysis in the Presence of Structural Breaks

Using the Gregory and Hansen (1996a; 1996b) tests, we tested for a bivariate cointegration relationship between oil prices and

**Table 2: Unit root tests for oil prices (WTI)**

Test	Whole sample		Pre-crisis		Crisis		Post-crisis	
	Levels	1 <sup>st</sup> diff.	Levels	1 <sup>st</sup> diff.	Levels	1 <sup>st</sup> diff.	Levels	1 <sup>st</sup> diff.
ADF unit root test								
Exogenous <sup>a</sup>	CLT	CLT	C	CLT	C	C	C	C
Lag length <sup>c</sup>	1	0	3	2	0	0	1	0
T-Stat.	-1.792	-28.098	0.061	-14.208	-0.359	-7.757	-1.422	-15.028
Test crit. values <sup>c</sup> : 1% level	-3.966	-3.966	-3.439	-3.971	-3.508	-3.509	-3.447	-3.447
Test crit. values <sup>c</sup> : 5% level	-3.414	-3.414	-2.865	-3.416	-2.896	-2.896	-2.869	-2.869
PP unit root test								
Exogenous <sup>b</sup>	CLT	CLT	C	CLT	C	C	C	C
Bandwidth <sup>d</sup>	14	12	13	17	4	4	6	2
Adj. t-Stat.	-2.296	-29.186	0.443	-22.696	-0.660	-7.939	-1.344	-15.011
Asymptotic crit. values <sup>c</sup> : 1% level	-3.966	-3.966	-3.439	-3.971	-3.508	-3.509	-3.447	-3.447
Asymptotic crit. values <sup>c</sup> : 5% level	-3.414	-3.414	-2.865	-3.416	-2.896	-2.896	-2.869	-2.869
KPSS unit root test								
Exogenous <sup>b</sup>	CLT	CLT	CLT	CLT	CLT	CLT	CLT	CLT
Bandwidth <sup>f</sup>	26	14	22	14	7	4	15	4
LM Stat.	0.3054	0.0527	0.5704	0.0405	0.2633	0.1408	0.5033	0.0440
Asymptotic crit. values <sup>c</sup> : 1% level	0.2160	0.2160	0.2160	0.2160	0.2160	0.2160	0.2160	0.2160
Asymptotic crit. values <sup>c</sup> : 5% level	0.1460	0.1460	0.1460	0.1460	0.1460	0.1460	0.1460	0.1460

CLT: Constant, Linear Trend, C indicates Constant, <sup>a</sup>Model selection is based on Schwarz information criterion, <sup>b</sup>Model selection is based on Newey–West bandwidth, <sup>c</sup>Lag length selection is based on Schwarz information criterion, maxlag=22, <sup>d</sup>The bandwidth selection is defined by using Bartlett kernel, <sup>e</sup>MacKinnon (1996), <sup>f</sup>Model selection is based on Newey–West bandwidth using Bartlett kernel. WTI: West Texas intermediate, ADF: Augmented dickey fuller, PP: Phillips-perron, KPSS: Kwiatkowski–Phillips–Schmidt–Shin

**Table 3: Unit root tests for STLFSI**

Test	Whole sample		Pre-crisis		Crisis		Post-crisis	
	Levels	1 <sup>st</sup> diff.	Levels	1 <sup>st</sup> diff.	Levels	1 <sup>st</sup> diff.	Levels	1 <sup>st</sup> diff.
ADF unit root test								
Exogenous <sup>a</sup>	C	CLT	C	CLT	CLT	C	CLT	NONE
Lag length <sup>c</sup>	7	6	1	0	1	0	1	0
T-Stat.	-2.377	-13.275	-1.827	-22.308	-2.266	-6.318	-5.272	-14.758
Test crit. values <sup>c</sup> : 1% level	-3.436	-3.966	-3.439	-3.971	-4.070	-4.070	-3.982	-2.571
Test crit. values <sup>c</sup> : 5% level	-2.864	-3.414	-2.865	-3.416	-3.464	-3.464	-3.422	-1.942
PP unit root test								
Exogenous <sup>b</sup>	CLT	CLT	C	CLT	CLT	CLT	CLT	NONE
Bandwidth <sup>d</sup>	6	3	2	4	5	4	5	8
Adj. T-Stat.	-3.115	-25.206	-1.681	-22.229	-2.095	-6.397	-5.414	-15.226
Asymptotic crit. values <sup>c</sup> : 1% level	-3.966	-3.966	-3.439	-3.971	-4.068	-4.070	-3.982	-2.571
Asymptotic crit. values <sup>c</sup> : 5% level	-3.414	-3.414	-2.865	-3.416	-3.463	-3.464	-3.422	-1.942
KPSS unit root test								
Exogenous <sup>b</sup>	CLT	CLT	CLT	CLT	C	C	CLT	CLT
Bandwidth <sup>f</sup>	26	5	21	0	7	4	15	8
LM Stat	1.3792	0.0268	0.6042	0.0620	0.8146	0.0679	0.3727	0.1688
Asymptotic crit. values <sup>c</sup> : 1% level	0.7390	0.2160	0.2160	0.2160	0.7390	0.7390	0.2160	0.2160
Asymptotic crit. values <sup>c</sup> : 5% level	0.4630	0.1460	0.1460	0.1460	0.4630	0.4630	0.1460	0.1460

CLT: Constant, Linear Trend, C indicates Constant, <sup>a</sup>Model selection is based on Schwarz information criterion, <sup>b</sup>Model selection is based on Newey–West Bandwidth, <sup>c</sup>Lag length selection is based on Schwarz information criterion, maxlag=22, <sup>d</sup>The bandwidth selection is defined by using Bartlett kernel, <sup>e</sup>MacKinnon (1996), <sup>f</sup>Model selection is based on Newey–West bandwidth using Bartlett kernel. STLFSI: St. Louis Fed Financial Stress Index, ADF: Augmented dickey fuller, PP: Phillips-perron, KPSS: Kwiatkowski–Phillips–Schmidt–Shin

financial stress in the US markets. Results of the residual-based tests for cointegration in models with regime shifts are reported in Table 6. In almost the cases, the hypothesis of cointegration with a structural break is not supported at better than 1% significance level.

Taking into account the structural break in the cointegrating relationship between oil prices and financial stress, the adjustment of the oil price to changes in the financial stress in the long-run is explored allowing for the possibility of asymmetries in the error

correction process. To deal with the issue of statistical inference in a cointegrated system with structural breaks, both TAR and MTAR models (Enders and Siklos, 2001) are used.

#### 4.4. Results of the Threshold Cointegration Analysis

From the linear cointegration tests, the price transmission mechanism between oil prices and financial stress index may be asymmetric. To investigate this possibility, it is necessary to go further than the usual concept of cointegration in order to allow for asymmetric cointegration and thus asymmetric price

**Table 4: Results of the Johansen cointegration tests**

Test	Specification	Lag	Statistic	Critical value (%)		
				10	5	1
Johansen $\lambda_{max}$						
$r=1$	Trend	5	5.472	10.49	12.25	16.26
$r=0$	Trend	5	21.152**	16.85	18.96	23.65
$r=1$	Constant	5	4.199	7.52	9.24	12.97
$r=0$	Constant	5	9.511	13.75	15.67	20.2
$r=1$	None	5	4.098	6.5	8.18	11.65
$r=0$	None	5	9.511	12.91	14.9	19.19
Johansen $\lambda_{trace}$						
$r \leq 1$	Trend	5	5.472	10.49	12.25	16.26
$r=0$	Trend	5	26.624**	22.76	25.32	30.45
$r \leq 1$	Constant	5	4.199	7.52	9.24	12.97
$r=0$	Constant	5	13.71	17.85	19.96	24.6
$r \leq 1$	None	5	4.098	6.5	8.18	11.65
$r=0$	None	5	13.609	15.66	17.95	23.52

$r$  is the number of cointegrating vectors. The critical values are from Enders (2004). \*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% levels, respectively

**Table 5: Results of the linear (Engle-Granger) cointegration tests**

Variable	Whole sample	Pre-crisis	In-crisis	Post-crisis
First step				
$\xi_0$	51.139*** [61.64]	36.118*** [82.690]	110.160*** [27.518]	72.039*** [45.268]
$\xi_1$	-13.067*** [-15.74]	-22.169*** [-30.900]	-12.511*** [-7.266]	-9.355*** [-6.338]
Second step				
Lags ( $\rho$ )	9	3	1	1
$\rho$	-0.005* [-1.975]	-0.017** [-2.435]	-0.047* [-1.763]	-0.006 [-0.939]
$\phi_1$	0.253*** [8.617]	0.185*** [4.887]	0.235** [2.228]	0.262*** [5.245]
$\phi_2$	-0.104*** [-3.439]	-0.070* [-1.837]	—	—
$\phi_3$	0.147*** [4.820]	0.089** [2.377]	—	—
$\phi_4$	-0.098** [-3.195]	—	—	—
$\phi_5$	0.043 [1.402]	—	—	—
$\phi_6$	-0.041 [0.186]	—	—	—
$\phi_7$	-0.012 [-0.411]	—	—	—
$\phi_8$	0.009 [0.329]	—	—	—
$\phi_9$	0.058* [1.980]	—	—	—
AIC	5332.006	2966.987	540.394	1723.438
BIC	5387.784	2989.806	547.792	1735.267
$Q_{LB}(4)$	0.998	0.793	0.124	0.87
$Q_{LB}(8)$	1.000	0.171	0.429	0.815
$Q_{LB}(12)$	0.127	0.115	0.263	0.518

The numbers in the brackets are t-values. \*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% levels, respectively

**Table 6: Results of Gregory-Hansen cointegration test**

Period	Level shift model C				Level shift model with trend C/T			
	t-Stat.	$t_0$	Lag	Date	t-Stat.	$t_0$	Lag	Date
Whole sample	-3.883	565	9	2004:10:22	-3.853	565	9	2004:10:22
Pre-crisis	-4.108	568	3	2004:11:12	-4.741	568	3	2004:11:12
In-crisis	-3.133	60	0	2008:09:19	-5.043	71	3	2008:12:05
Post-crisis	-4.739	293	1	2014:11:07	-4.817	293	1	2014:11:07
Period	Regime shift model C/S				Regime shift model with trend C/S/T			
	t-Stat.	$t_0$	Lag	Date	t-Stat.	$t_0$	Lag	Date
Whole sample	-5.170	795	12	2009:03:20	-3.888	565	9	2004:10:22
Pre-crisis	-4.917	546	1	2004:06:11	-4.48	565	1	2004:10:22
In-crisis	-5.728	71	3	2008:12:05	-3.288	60	0	2008:09:19
Post-crisis	-5.069	293	1	2014:11:07	-4.822	293	1	2014:11:07

t-Stat. indicate smallest t-statistics using Gregory-Hansen cointegration test among possible break points. Three asterisks \*\*\* (resp. \*\*, \*) denote rejection of the null hypothesis at the 1% (resp. 5%, 10%) significance level.  $t_0$  denotes the break point corresponding to the smallest t-statistic.

transmission. We conduct a nonlinear cointegration analysis by using the threshold auto-regression models. A total of four models are considered in this study. They are TAR with  $\tau=0$ , consistent TAR (C.TAR) with  $\tau$  estimated, M-TAR with  $\tau=0$  and consistent M-TAR (C.M-TAR) with  $\tau$  estimated.

To address possible serial correlation in the residual series, we select an appropriate lag by specifying a maximum lag of 16. We use AIC, BIC and Ljung-Box  $Q$  statistics for diagnostic analyses on the residuals. In most cases, the value of the threshold  $\tau$  is unknown and has to be estimated along the values of  $\rho_1$  and  $\rho_1$ .

We follow the Chan's (1993) method to estimate the threshold values for consistent TAR and M-TAR models.

The empirical results of the threshold cointegration tests with an unknown threshold value using TAR, M-TAR and their consistent counterparts are reported in Tables 7 and 8. The value is the optimal threshold for the indicator function. Under these conditions, we can reject the null hypothesis of threshold cointegration ( $\rho_1 = \rho_2 = 0$ ) for all considered cases. This means that there exists a cointegrating relationship between oil prices and financial stress index. Since WTI is cointegrated with STLFSI using the consistent TAR or M-TAR models, we examine whether their adjustment coefficients are different across positive and negative errors. This procedure is achieved by verifying the existence of an asymmetric cointegration, i.e., testing the null assumption of  $\rho_1 = \rho_2$ . Notice that the asymmetry test only makes sense when the two previous tests reject the null hypothesis. That is, if the  $\rho_1$  coefficients estimated for the threshold are significantly different from zero, then the regression is nontrivial and testing for symmetry makes all the sense.

Based on the "Principle of Parsimony," AIC, BIC and Ljung-Box  $Q$  statistics suggest that the most applicable model for variables' adjustment to long-run equilibrium is the M-TAR model with consistent threshold value for the whole sample, while the consistent TAR model is the best one for the in-crisis and post-crisis periods.

It turns out that different lag specifications in the models have little impact of the final threshold values selected. The variation of the SSE by threshold value for consistent M-TAR model with a lag of twelve is presented in Figure 2.

The lowest SSE for the consistent M-TAR model is 6851.566 at the threshold value of  $-1.093$ . Similarly, the best threshold value with the lowest SSE is estimated to be 24.604 for the consistent TAR model. Finally, while the four nonlinear threshold cointegration models have similar results (Table 7), the consistent M-TAR model has the lowest AIC statistic of 5301.544 and BIC statistic of 5377.438, and therefore, is deemed to be the best.

As shown in Tables 7 and 8, we found limited evidence of asymmetric price transmission between oil prices and financial stress. Therefore, oil prices became cointegrated with the STLFSI, the adjustment mechanism is asymmetric and the speed of adjustment to the equilibrium is different when the last equilibrium error has different signs. This means that the change in the equilibrium error has a different impact on the adjustment speed to the new equilibrium.

Focusing on the results from the consistent M-TAR model, the F-test for the null hypothesis of no cointegration has a statistic of 6.165 and it is highly significant at the 1% level. Thus, the oil prices and financial stress index are cointegrated with threshold adjustment. Furthermore, the F-statistic for the null hypothesis of symmetric price transmission has a value of 8.746 and it is also significant at the 1% level. Therefore, the adjustment process is

asymmetric when WTI and STLFSI adjust to achieve the long-term equilibrium.

When considering the in-crisis period and focusing on the results from the consistent TAR model, the F-test for the null assumption of no cointegration has a statistic of 3.235 and it is statistically significant at the 5% level. Thus, WTI and STLFSI are cointegrated with threshold adjustment. In addition, the F-statistic for the null assumption of symmetric price transmission has a value of 3.277 and it is statistically significant at the 10% level. Therefore, during the global financial crisis period, the adjustment process is asymmetric when WTI and STLFSI adjust to achieve the long-term equilibrium.

Focusing on the findings from the consistent TAR model for the post-crisis period, the F-test for the null hypothesis of no cointegration has a statistic of 2.32 and it is significant at the 10% level. Therefore, the oil prices and financial stress index are cointegrated with threshold adjustment during the post-crisis period. Furthermore, the F-statistic for the null hypothesis of symmetric price transmission has a value of 3.753 and it is also significant at the 10% level. Thus, during the post-crisis period, the adjustment process is asymmetric when WTI and STLFSI adjust to achieve the long-term equilibrium.

#### 4.5. Results of the Error Correction Model

Given the nonlinear threshold cointegration results, the final step in our analysis is to proceed with the asymmetric error correction model in order to investigate the movement of the oil price and STLFSI index series in a long-run equilibrium relationship. For the whole sample period, the results of our estimations of the consistent M-TAR error correction models are illustrated in Table 9. Diagnostic analyses on the residuals with AIC, BIC and Ljung-Box  $Q$  statistics select a lag of eight for the model. The consistent M-TAR model is the best from the threshold cointegration analyses and the error correction terms are constructed using Eq. (17) and Eq. (19). Results show that WTI is cointegrated with STLFSI index and it also exhibits asymmetric adjustments. Besides, the short-term equilibrium adjustment process mainly occurs with STLFSI-index since  $\delta^+ = \delta^-$ . Moreover, there are three situations to reduce the price deviations between the two variables if they are cointegrated (Chen et al., 2013). Given the case STLFSI-index price is larger than WTI price, there are three situations to reduce the price deviations: (i) STLFSI-index price goes down and WTI price goes up; (ii) STLFSI-index price goes down and WTI price goes down as well, but STLFSI-index price drops more; (iii) STLFSI-index price goes up and WTI price goes up, but STLFSI-index price increases less.

In our empirical results, for regimes with positive shocks (STLFSI-index price is higher than WTI price), the adjustment coefficient for STLFSI-index is 0.0001 and -0.003 for WTI, which means that, in the next period, WTI price will go up and STLFSI-index price will go down, and thus, the price deviation will decrease. For regimes with negative shocks (STLFSI-index price is lower than WTI price), the adjustment coefficient for STLFSI-index is  $-0.0002$  and  $-0.018$  for WTI, which means that, in the next period, WTI price will go down and STLFSI-index price will go down as well,

Table 7: Results of the nonlinear (threshold) cointegration tests

Variable	Whole sample				Pre-crisis			
	TAR	C.TAR	MTAR	C.MTAR	TAR	C.TAR	MTAR	C.MTAR
Lags ( $\rho$ )	12	12	12	12	7	7	7	7
Threshold ( $\tau$ )	0.000	24.604	0.000	-1.093	0.000	-8.237	0.000	1.22
$\rho_1$	-0.005 [-1.637]	-0.008*** [-2.645]	0.001 [0.117]	0.001 [0.126]	-0.009 [-0.953]	-0.008 [-0.948]	-0.014 [-1.475]	-0.006 [-0.405]
$\rho_2$	-0.004 [-0.965]	0.001 [0.217]	-0.01*** [-2.758]	-0.017*** [-3.510]	-0.026** [-2.323]	-0.03** [-2.504]	-0.018* [-1.665]	-0.019** [-2.340]
$\varphi_1$	0.258*** [8.780]	0.260*** [8.865]	0.239*** [7.764]	0.226*** [7.260]	0.172*** [4.525]	0.172*** [4.531]	0.173*** [4.519]	0.168*** [4.349]
$\varphi_2$	-0.094** [-3.119]	-0.092** [-3.055]	-0.098** [-3.240]	-0.099** [-3.277]	-0.064* [-1.679]	-0.064* [-1.668]	-0.064* [-1.662]	-0.064* [-1.668]
$\varphi_3$	0.139*** [4.602]	0.141*** [4.645]	0.139*** [4.587]	0.140*** [4.650]	0.089** [2.323]	0.089** [2.321]	0.089** [2.326]	0.089** [2.337]
$\varphi_4$	-0.102*** [-3.340]	-0.100** [-3.287]	-0.100** [-3.284]	-0.099** [-3.274]	0.048 [1.249]	0.049 [1.259]	0.048 [1.245]	0.046 [1.198]
$\varphi_5$	0.044 [1.429]	0.045 [1.468]	0.049 [1.589]	0.053* [1.720]	0.032 [0.825]	0.033 [0.854]	0.033 [0.849]	0.032 [0.843]
$\varphi_6$	-0.041 [-1.320]	-0.039 [-1.258]	-0.039 [-1.290]	-0.038 [-1.248]	-0.075* [-1.949]	-0.074* [-1.936]	-0.074* [-1.937]	-0.076** [-1.984]
$\varphi_7$	-0.013 [-0.411]	-0.011 [-0.373]	-0.012 [-0.383]	-0.014 [-0.451]	-0.070* [-1.834]	-0.068* [-1.790]	-0.069* [-1.826]	-0.069* [-1.823]
$\varphi_8$	0.021 [0.669]	0.021 [0.701]	0.016 [0.525]	0.018 [0.582]	-	-	-	-
$\varphi_9$	0.056* [1.832]	0.058* [1.887]	0.056* [1.842]	0.056* [1.842]	-	-	-	-
$\varphi_{10}$	-0.043 [-1.428]	-0.041 [-1.356]	-0.045 [-1.493]	-0.044 [-1.456]	-	-	-	-
$\varphi_{11}$	-0.083** [-2.732]	-0.081** [-2.674]	-0.079** [-2.623]	-0.080** [-2.671]	-	-	-	-
$\varphi_{12}$	0.089** [3.037]	0.091** [3.112]	0.088** [2.995]	0.089** [3.061]	-	-	-	-
Total obs.	1177	1177	1177	1177	709	709	709	709
Coint obs.	1164	1164	1164	1164	701	701	701	701
AIC	5310.332	5306.851	5306.280	5301.544	2950.720	2949.870	2952.054	2951.276
BIC	5386.226	5382.745	5382.174	5377.438	2996.246	2995.395	2997.579	2996.801
$Q_{LB}(4)$	0.999	0.999	0.999	0.998	1.000	1.000	1.000	1.000
$Q_{LB}(8)$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$Q_{LB}(12)$	1.000	1.000	1.000	1.000	0.880	0.877	0.880	0.842
No CI: $\Phi$	1.794 [0.167]	3.521** [0.030]	3.805** [0.023]	6.165*** [0.002]	3.061** [0.047]	3.485** [0.031]	2.397* [0.092]	2.784* [0.062]
$(H_0: \rho_1 = \rho_2 = 0)$	0.030 [0.861]	3.475* [0.063]	4.041** [0.045]	8.746*** [0.003]	1.382 [0.240]	2.224 [0.136]	0.064 [0.800]	0.833 [0.362]
No APT: $F$								
$(H_0: \rho_1 = \rho_2)$								

\*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% levels, respectively. Numbers in parentheses are P values. Numbers in brackets are t-values

Table 8: Results of the nonlinear (threshold) cointegration tests (continued)

Variable	In-crisis				Post-crisis			
	TAR	C.TAR	MTAR	C.MTAR	TAR	C.TAR	MTAR	C.MTAR
Lags ( $\rho$ )	4							
Threshold ( $\tau$ )	0.000	-27.000	0.000	-6.15	0.000	17.019	0.000	-1.395
$\rho_1$	-0.043 [-1.161]	-0.019 [-0.627]	-0.013 [-0.351]	-0.029 [-0.999]	-0.02* [-1.961]	-0.027** [-2.153]	-0.006 [-0.674]	-0.01 [-1.398]
$\rho_2$	-0.039 [-0.915]	-0.127** [-2.469]	-0.087** [-2.238]	-0.137** [-2.116]	0.001 [0.205]	0.0004 [-0.061]	-0.006 [-0.654]	0.006 [0.529]
$\phi_1$	0.309** [2.794]	0.245** [2.350]	0.228** [2.169]	0.227** [2.165]	0.264*** [5.307]	0.265*** [5.314]	0.262*** [5.233]	0.262*** [5.258]
$\phi_2$	-0.129-1.146	-	-	-	-	-	-	-
$\phi_3$	0.272**2.438	-	-	-	-	-	-	-
$\phi_4$	-0.180 [-1.615]	-	-	-	-	-	-	-
Total obs.	87	87	87	87	381	381	381	381
Coint obs.	82	85	85	85	379	379	379	379
AIC	523.597	539.063	540.363	540.021	1722.419	1721.674	1725.438	1724.074
BIC	540.444	548.834	550.133	549.792	1738.169	1737.424	1741.188	1739.824
$Q_{LB}(4)$	0.996	0.191	0.224	0.378	0.906	0.933	0.87	0.831
$Q_{LB}(8)$	0.993	0.551	0.601	0.785	0.816	0.845	0.815	0.826
$Q_{LB}(12)$	0.942	0.412	0.329	0.574	0.53	0.588	0.518	0.525
No CI: $\Phi$	1.058 [0.352]	3.235** [0.044]	2.564* [0.083]	2.739* [0.071]	1.946 [0.1442]	2.32* [0.099]	0.439 [0.645]	1.119 [0.328]
$(H_0 : \rho_1 = \rho_2 = 0)$	0.004 [0.950]	3.277* [0.074]	1.983 [0.163]	2.321 [0.131]	3.007* [0.084]	3.753* [0.053]	0.0002 [0.995]	1.356 [0.245]
No APT: $F$								
$(H_0 : \rho_1 = \rho_2)$								

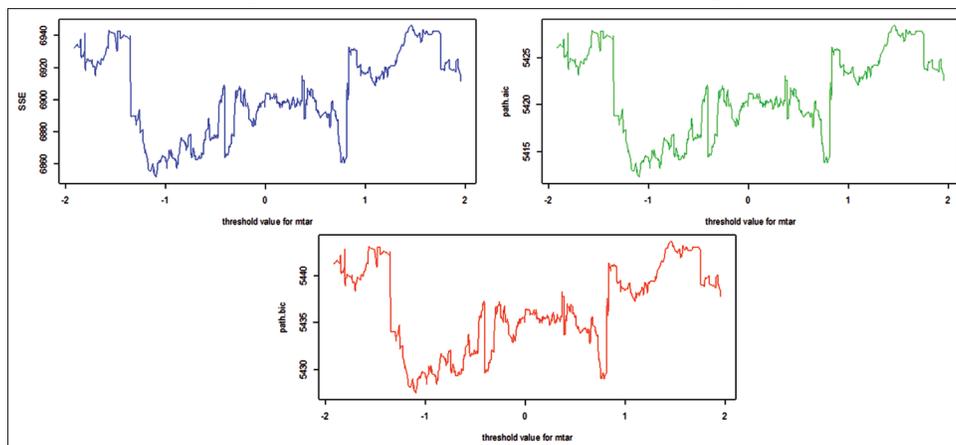
\*\*\*, \*\* and \* denote statistical significance at the 1%, 5% and 10% levels, respectively. Numbers in parentheses are P values. Numbers in brackets are t-values

but WTI drops more and thus the price deviation will decrease. The adjusted R-squared value is 0.243 for the STLFSI-index and 0.131 for WTI. Moreover, the AIC and BIC statistics for WTI are both larger than those for the STLFSI-index. This means that the model specification is better fitted on the WTI price.

Using the estimation results of the asymmetric ECM with nonlinear threshold cointegration, we also conduct the hypothesis testing described in Section 3 (paragraph 3.4). The hypotheses of Granger causality between the series are assessed with F-tests. The F-statistic of 5.724 reveals that STLFSI does Granger cause WTI. Besides, the F-statistic of 4.941 indicates that WTI does Granger cause STLFSI. This indicates that, in the short-term, both variables affect each other. Similarly, the F-statistic of 16.899 for STLFSI discloses that the lagged index series have significant impacts on its own index. Furthermore, the F-statistic of 5.134, for WTI, reveals that the lagged price series have significant impacts on its own price. Thus, in the short term, STLFSI and WTI have been evolving more dependently.

A number of hypotheses are examined for asymmetric price transmission. The first one is the distributed lag asymmetric effect. In each price equation, the equality of the corresponding positive and negative coefficients for each of the eight lags is tested; in total, there are sixteen F-tests for this hypothesis. It turns out that an important number of them is statistically significant and distributed lag asymmetric effect does exist. Furthermore, the cumulative asymmetric effects are also examined. The largest F-statistic is 9.963 but only one of the four statistics is statistically significant at the 1% level. Thus, cumulative effects are asymmetric. The fourth examined asymmetry is the momentum equilibrium adjustment path asymmetries ( $H_{07} : \delta^+ = \delta^-$ ). For STLFSI, the F-statistic is 1.706 with a P = 0.192. The point estimates of the coefficients for the error correction terms are 0.0001 with a t-value of 0.668 for positive error correction term and -0.0002 with a t-value of -0.82 for the negative one. In contrast, for WTI price, the F-statistic is 7.952 with a P = 0.005. Thus, there is momentum equilibrium adjustment asymmetry. The point estimates are -0.003 with a t-value of -0.746 for positive deviations and -0.018 with a t-value of -3.405 for negative deviations. The magnitude suggests that in the short term the WTI responds to the positive deviations by 0.3% in a week but by 1.8% to negative deviations. Measured in response time, positive and negative deviations take, respectively, 333.333 and 55.556 weeks to be fully digested. Therefore, in the short-term, WTI has a much faster reaction to negative deviations from long-term equilibrium than positive deviations.

Further findings for the in-crisis period are reported in Table 9. Diagnostic analyses on the residuals with AIC, BIC and Ljung-Box  $Q$  statistics select a lag of four for the model. The consistent TAR model is the best from the threshold cointegration analyses and the error correction terms are constructed using Eq. (16) and Eq. (19). During the global financial crisis period, results show that WTI is cointegrated with STLFSI index and it also exhibits asymmetric adjustments. Besides, for regimes with positive shocks, the adjustment coefficient for STLFSI-index is 0.004 and -0.036 for WTI, which means that, in the next period, WTI price will go up and STLFSI-index price will go down, and thus, the

**Figure 2:** Threshold value for M-TAR (whole sample)

price deviation will decrease. For regimes with negative shocks, the adjustment coefficient for STLFSI-index is  $-0.002$  and  $-0.076$  for WTI, which means that, in the next period, WTI price will go down and STLFSI-index price will go down as well, but WTI drops more and thus the price deviation will decrease. The adjusted R-squared value is 0.337 for the STLFSI-index and 0.208 for WTI price. Likewise, the AIC and BIC statistics for WTI are both larger than those for the STLFSI-index, which means that the model specification is better fitted on the WTI price during the in-crisis period. Furthermore, the F-statistic of 1.811 reveals that STLFSI does Granger cause WTI. Moreover, the F-statistic of 3.098 indicates that WTI does Granger cause STLFSI. This indicates that, in the short-term, both variables affect each other during the crisis period. In the same way, the F-statistic of 2.34 for STLFSI discloses that the lagged index series have significant impacts on its own index. Additionally, the F-statistic of 0.781, for WTI, reveals that the lagged price series have insignificant impacts on its own price. Accordingly, in the short term, STLFSI and WTI have been evolving more dependently during the in-crisis period.

The results for hypothesis description are also reported in Table 9. In each price equation, the equality of the corresponding positive and negative coefficients for each of the four lags is tested; in total, there are eight F-tests for this assumption. It turns out that a few numbers of them is statistically significant and the distributed lag asymmetric effect does exist. Furthermore, the cumulative asymmetric effects are not statistically significant and thus, cumulative effects are symmetric. In addition, the momentum equilibrium adjustment path asymmetry is examined. For STLFSI, the F-statistic is 1.893 with a  $P = 0.174$ . The point estimates of the coefficients for the error correction terms are 0.004 with a t-value of 1.889 for positive error correction term and  $-0.002$  with a t-value of 0.502 for the negative one. On the contrary, for WTI price, the F-statistic is 0.377 with a  $P = 0.174$ . Thus, there is no momentum equilibrium adjustment asymmetry.

The estimation and diagnostic results for the post-crisis period are reported in Table 9. Diagnostic analyses on the residuals with AIC, BIC and Ljung-Box Q statistics select a lag of five for the model. The consistent TAR model is the best from the threshold cointegration analyses and the error correction terms

are constructed using Eq. (16) and Eq. (19). During the post-crisis period, results show that WTI is cointegrated with STLFSI index and it also exhibits asymmetric adjustments. Besides, for regimes with positive shocks, the adjustment coefficient for STLFSI-index is  $-0.0003$  and  $-0.031$  for WTI, which means that, in the next period, WTI price will go up and STLFSI-index price will go down, and thus, the price deviation will decrease. For regimes with negative shocks, the adjustment coefficient for STLFSI-index is  $-0.0001$  and 0.002 for WTI, which means that, in the next period, WTI price will go down and STLFSI-index price will go down as well, but WTI drops more and thus the price deviation will decrease. The AIC and BIC statistics for WTI are both larger than those for the STLFSI-index, which means that the model specification is better fitted on the WTI price during the post-crisis period. Furthermore, the F-statistic of 0.953 reveals that STLFSI does not Granger cause WTI. Moreover, the F-statistic of 0.736 indicates that WTI does not Granger cause STLFSI. This indicates that, in the short-term, both variables do not affect each other during the post-crisis period. Also, the F-statistic of 2.938 for STLFSI discloses that the lagged index series have significant impacts on its own index. Additionally, the F-statistic of 3.869, for WTI, reveals that the lagged price series have significant impacts on its own price. Accordingly, in the short term, STLFSI and WTI have been evolving more independently during the post-crisis period.

Finally, the results for hypothesis description are summarized in Table 9. First, they indicate the existence of some distributed lag asymmetric effects. Second, the cumulative effects are asymmetric. Third, there is a momentum equilibrium adjustment path asymmetry. For STLFSI, the F-statistic is 0.189 with a  $P = 0.664$ . The point estimates of the coefficients for the error correction terms are  $-0.0003$  with a t-value of  $-0.619$  for positive error correction term and  $-0.0001$  with a t-value of  $-0.266$  for the negative one. In contrast, for WTI price, the F-statistic is 3.274 with a  $P = 0.071$ . Furthermore, the point estimates are  $-0.031$  with a t-value of  $-1.947$  for positive deviations and 0.002 with a t-value of 0.216 for negative deviations. The magnitude suggests that in the short term the WTI responds to the positive deviations by 3.1% in a week but by 0.2% to negative deviations. Measured in response time, positive and negative deviations take, respectively, 32.26 and

Table 9: Results of the asymmetric error correction models with threshold cointegration

Variable	Whole sample (C.M-TAR; lag=8)			In-crisis (C. TAR; lag=4)			Post-crisis (C. TAR; lag=5)		
	STLFSI	WTI	WTI	STLFSI	WTI	WTI	STLFSI	WTI	WTI
	Coefficient	t-statistic	t-statistic	Coefficient	t-statistic	t-statistic	Coefficient	t-statistic	t-statistic
$\theta$	0.001	0.144	-0.112	0.065	0.713	1.918	0.015	1.318	-0.65*
$\alpha_1^+$	0.433***	9.41	2.649***	0.565***	3.098	4.715	-0.033	-0.341	5.565**
$\alpha_2^+$	0.058	1.198	-6.475***	0.095	0.47	-7.125**	0.042	0.431	0.301
$\alpha_3^+$	0.076	1.528	1.188	0.192	0.953	2.824	-0.089	-0.905	1.865
$\alpha_4^+$	-0.161***	-3.191	-2.901***	-0.345*	-1.744	-3.259	0.051	0.523	-2.082
$\alpha_5^+$	-0.309***	-6.015	3.673***	3.42	-	-	0.055	0.563	-0.216
$\alpha_6^+$	0.272***	5.234	-0.75	-0.691	-	-	-	-	-
$\alpha_7^+$	-0.282***	-5.41	0.988	0.908	-	-	-	-	-
$\alpha_8^+$	0.004	0.07	-3.808***	-3.566	-	-	-	-	-
$\alpha_1^-$	0.135**	2.211	-1.612	-1.265	-0.873	0.463	0.419***	4.117	1.274
$\alpha_2^-$	-0.093	-1.553	-0.52	-0.415	0.069	-3.415	0.016	0.151	0.25
$\alpha_3^-$	0.105*	1.743	-0.912	-0.727	1.17	0.972	0.141	1.352	-7.328**
$\alpha_4^-$	-0.018	-0.311	2.028*	1.652	-0.304	6.586*	0.089	0.861	-1.383
$\alpha_5^-$	0.162***	2.78	-2.423**	-1.993	-	-	0.011	0.111	3.441
$\alpha_6^-$	-0.012	-0.213	-2.411**	-2.005	-	-	-	-	-
$\alpha_7^-$	0.065	1.171	0.215	0.185	-	-	-	-	-
$\alpha_8^-$	0.163***	2.957	-0.517	-0.448	-	-	-	-	-
$\beta_1^+$	0.008***	2.697	0.107*	1.842	0.025	1.59	0.001	0.375	0.195*
$\beta_2^+$	-0.003	-1.119	-0.005	-0.095	-0.005	-0.3	-0.006**	-2.011	0.17*
$\beta_3^+$	-0.005	-1.628	0.112*	1.945	-0.04**	2.413	0.002	0.532	0.057
$\beta_4^+$	-0.004	-1.276	0.043	0.752	-0.028*	-1.701	0.002	0.593	0.119
$\beta_6^+$	0.0003	0.12	0.033	0.574	-	-	-0.001	-0.345	0.056
$\beta_7^+$	-0.004	-1.549	-0.096*	-1.691	-	-	-	-	-
$\beta_7^+$	0.003	0.992	0.048	0.845	-	-	-	-	-

(Contd...)



Table 9: (Continued)

Variable	Whole sample (C.M-TAR; lag=8)			In-crisis (C. TAR; lag=4)			Post-crisis (C. TAR; lag=5)			
	STLFSI	WTI	WTI	STLFSI	WTI	WTI	STLFSI	WTI	WTI	
	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	
$H_{03} : \alpha_{x5}^+ = \alpha_{x5}^- = 0$	27.526***	0.000	10.562***	0.001	-	-	0.074	0.786	0.431	0.512
$H_{03} : \alpha_{x6}^+ = \alpha_{x6}^- = 0$	9.927***	0.002	0.776	0.379	-	-	-	-	-	-
$H_{03} : \alpha_{x7}^+ = \alpha_{x7}^- = 0$	15.528***	0.000	0.177	0.674	-	-	-	-	-	-
$H_{03} : \alpha_{x8}^+ = \alpha_{x8}^- = 0$	3.300*	0.070	3.213*	0.073	-	-	-	-	-	-
$H_{04} : \beta_{y1}^+ = \beta_{y1}^- = 0$	9.808***	0.002	2.840*	0.092	1.985	0.164	1.304	0.254	1.493	0.223
$H_{04} : \beta_{y2}^+ = \beta_{y2}^- = 0$	2.699	0.101	2.204	0.138	0.000	0.985	2.581	0.109	4.622**	0.032
$H_{04} : \beta_{y3}^+ = \beta_{y3}^- = 0$	13.022***	0.000	1.104	0.294	11.226***	0.001	0.008	0.930	0.519	0.472
$H_{04} : \beta_{y4}^+ = \beta_{y4}^- = 0$	0.577	0.448	1.014	0.314	0.52	0.474	0.433	0.511	2.451	0.118
$H_{04} : \beta_{y5}^+ = \beta_{y5}^- = 0$	0.049	0.825	1.228	0.268	-	-	0.138	0.711	0.040	0.842
$H_{04} : \beta_{y6}^+ = \beta_{y6}^- = 0$	1.157	0.282	2.773*	0.096	-	-	-	-	-	-
$H_{04} : \beta_{y7}^+ = \beta_{y7}^- = 0$	1.321	0.251	0.974	0.324	-	-	-	-	-	-
$H_{04} : \beta_{y8}^+ = \beta_{y8}^- = 0$	21.007***	0.000	2.690	0.101	-	-	-	-	-	-
Cumulative asymmetric effect										
$H_{05} : \sum_{j=1}^p \alpha_{xj}^+ = \sum_{j=1}^p \alpha_{xj}^-$	9.963***	0.002	0.068	0.794	1.204	0.277	6.149**	0.014	1.012	0.315
$H_{06} : \sum_{j=1}^p \beta_{yj}^+ = \sum_{j=1}^p \beta_{yj}^-$	2.206	0.138	0.546	0.460	1.447	0.233	0.002	0.967	2.046	0.153
Equilibrium adjustment path asymmetry										
$H_{07} : \delta^+ = \delta^-$	1.706	0.192	7.952***	0.005	1.893	0.174	0.189	0.664	3.274*	0.071

$Q_{LB}$  (P) denotes the significance level for the Ljung-Box Q statistic, which tests serial correlation based on P autocorrelation coefficients. Numbers in parentheses are P values. \*\*\*, \*\*, and \* denote statistical significance at the 1%, 5% and 10% levels, respectively

500 weeks to be fully digested. Therefore, in the short-term, WTI has a much faster reaction to positive deviations from long-term equilibrium than negative deviations.

## 5. CONCLUDING REMARKS

In this article, we investigated the dynamic relationship between oil prices and financial stress over the period from December 31, 1993 to July 15, 2016. In particular, we focused on the linkages between those variables in both the long-run and short-run horizons under both the linear and nonlinear (threshold) cointegration framework. As an extension of preceding studies, we make use of the methodology developed by Enders and Siklos (2001), based on a nonlinear (threshold) cointegration model allowing for nonlinear adjustment to long-run equilibrium.

The results from the conventional linear cointegration approaches suggested that we can reject the null hypothesis of no cointegration. According to our empirical results, the null hypothesis of linear cointegration between oil prices and financial stress is rejected in favor of a threshold cointegration model in the sense that the short-term adjustments to the equilibrium are asymmetric depending on the deviation from the equilibrium. Thereafter, using the consistent TAR and MTAR specifications, we found evidence of asymmetry in the adjustment process to equilibrium. These findings would suggest the presence of a significant nonlinear behavior in the oil price-financial stress relationship. We find an asymmetric effect in the short-term adjustment process. Regimes with negative (below the threshold) changes of deviations adjust much quicker than regimes with positive (above the threshold) changes of deviations, especially during the crisis period. After incorporating the asymmetric adjustment and using a Granger causality test, we find a bi-directional causality between oil prices and financial stress index, indicating that these variables affect have been evolving more dependently in the short term, in particular during the crisis period. Our results also reveal the existence of three types of statistical significant asymmetries, namely the distributed lag asymmetric effect, the cumulative asymmetric effects, and the momentum equilibrium adjustment path asymmetries. Future research could extend the two-regime threshold cointegration model to three or more regimes.

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