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# Fractal Dimension Option Hedging Strategy Implementation during Turbulent Market Conditions in Developing and Developed Countries

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**ABSTRACT**

A hedging strategy is designed to increase the likelihood of desired financial outcomes. Market speculators hedge investment positions if they are worth protecting against potential negative outcomes of turbulent market conditions and effective hedging implementation can reduce the impact severity on the underlying investment since these negative scenarios cannot be avoided. This paper provides a solution for investors to implement a trading strategy to effectively manage turbulent market conditions (such as the COVID pandemic) by implementing an investment trading approach. The investment strategy includes an index held by the investor (long position) and uses a fractal dimension indicator to warn when liquidity or sentiment changes are imminent within financial markets. When the threshold is breached at a predetermined level, the investor will take this observation as a change in liquidity in the market and a hedging position is undertaken. This sequence of events triggers the implementation of a hedging strategy by entering a buy put option position. The fractal indicator was found to be effective when applied to four of the six tested indices in terms of cumulative returns, but also in effect increased the risk taken by the investor for all six indices. The conclusion was made that where the outcome was similar for each economy type, both had a scenario where two out of the three economies outperformed the underlying index and had one index not outperforming the underlying index. This comparison was done to establish whether the hedging strategy had a more promising application to a developing or developed economy type. The fractal indicator was found to be effective when applied to four of the six tested indices in terms of cumulative returns, but also in effect increased the risk taken by the investor for all six indices.

**Keywords:** Fractal Market Hypothesis, Hedging Strategy, Trading Strategy, Market Volatility, Covid-19
**JEL Classifications:** D53, D81, O16

## 1. INTRODUCTION

Investment strategies involve assembling portfolios based on the knowledge, insight, behaviour and perceived skill of the investor. The implementation of these investment approaches is driven by investor objectives, which are not limited to maximise the absolute return, outperforming the benchmark and minimise the portfolio variance (Crapo, 1999). Hedging strategies can be used in a conjunction manner with investment strategies to perfect the impact of negative price movement against the investor. Therefore, hedging strategies protect investors' portfolio values in volatile or uncertain market conditions. Derivative instruments in financial markets are usually used or implemented by an investor to limit losses brought about by volatile market conditions. By using financial derivatives, investors place a certain type of insurance on investment portfolios (Lewellen et al., 1977). The misalignment of single or even multiple investment objectives lubricates financial markets, giving rise to liquidity in efficient markets which includes but is not limited to risk, returns, disparate investment horizons. This misalignment is to the advantage of market participants; the ability to trade large asset volumes quickly by pooling large numbers of buyers and sellers who think and trade differently, without impairing the market price, is highly desirable (Vigna and Haberman, 2001; Borio and Lowe, 2002).

Stock prices rarely display mean-reverting behaviour and are more likely to follow a random walk behaviour. The mean-reverting behaviour is identifiable in their daily return value, which fluctuates randomly around zero. To obtain a mean-reverting scenario a portfolio of stocks or indexes can be synthesised to construct a co-integrated portfolio to obtain mean-reverting behaviour. This selection will also introduce stationarity among the portfolio or index. There are various statistical methods to identify stationarity within the constructed portfolio or on the applicable index where the Hurst exponent will be used to identify stationarity. By introducing a more hands-on approach to showcase the stationarity of a stock's price it can be represented as the price of a stock by $S(t)$, which has the characteristics of mean-reverting behaviour. Formally this behaviour can be described by the following Stochastic Differential Equation (SDE):

$$dS_t = \theta(\mu - S_t)dt + \sigma \cdot S_t dWt \quad (1)$$

SDEs can be formally described as a stochastic equation where one or more of the terms within the equation has to be a stochastic process. By using and implementing a stochastic process, investors can model phenomena within stock prices. Stochastic processes give investors the ability to model unstable stock prices.

The symbols identified in (8) are, therefore, $S_t$, $W_t$, $\theta$, $\mu$ and $\sigma$. Each symbol is the stock price at time $t$, respectively. The rate of reversion is given by $\theta$ to the mean of the stock price, the mean value of the stochastic process is shown by $\mu$ and lastly, the volatility of the stock is given by $\sigma$. By defining this SDE it is known that the variation of the price at $t$+1 is proportional to the difference between the price of the stock at time t and the mean of the stock. Lastly, it can also be assumed that the price variation has a higher probability of being positive (in a trending market) if the price of the stock is smaller than the mean.

The remainder of this paper proceeds as follows. Being an investment strategy, the fractal dimension indicator has been the focus of a fevered academic investigation, so literature on the subject is somewhat limited to generic option theory and performance measurement. The data (Section 3.1) used in this paper and the methodology (Section 3.2) create the fractal dimension indicator model that is provided in Section 3 and the results obtained are presented and discussed in Section 4 and Section 5 concludes this paper with recommendations for further research.

## 2. LITERATURE REVIEW

The literature review for this paper follows the approach and technique for identifying the fractal dimension of an underlying asset.

### 2.1. Tests of Non-stationarity on Stock Prices

Two of the most popular tests for non-stationarity are the Dickey-Fuller (DF) test and the Augmented Dickey-Fuller (ADF) test. The ADF is an extension of the Dickey-Fuller test, so first, the DF test is considered to better understand the ADF test. A simple model is shown as:

$$S_t = \rho \cdot S_{t-1} + \epsilon_t \quad (2)$$

$S_t$ can be described as varying stock prices at time $t$, $\rho$ is the coefficient and lastly $\epsilon_t$ is the error term within this simple model. The null hypothesis in this simple model can be given where $\rho = 1$. Given the circumstances here, both $S_t$ and $S_{t-1}$ are non-stationary and this, violates the central limit theorem (CLT) and an investor has to resort to the following process of first defining the first difference and the parameter $\delta$ as follows:

$$\Delta S_t = S_t - S_{t-1}, \text{ where } \delta = (\rho - 1) \quad (3)$$

The simple regression model can now more easily be written as:

$$\Delta S_t = \delta\, S_{t-1} + \epsilon_t \quad (4)$$

The DF tests will then test the null hypothesis where $\delta = 0$. The DF test and the logic behind this test can be interpreted in the following manner if it is shown that $S_t$ is stationary, the variable tends to behave in a manner that it reverts to a constant mean. This also shows a determinable trend that evolves within the stock price. Inherently this shows that larger values are more likely to follow smaller values and vice versa.

Second to last, this shows that the current price of the stock at time $t$ is a strong indicator of the next value at time $t$. And will inherently show a value where $\delta<0$. On the other hand, if it is found that $S_t$ is non-stationary, it is assumed that future price changes do not on the current price at time $t$. This would be described when the stock price is on a random walk.

Next, the ADF test is where a similar process is followed but in a more complex and complete model given by (5):

$$\Delta S_t = \alpha + \beta_t + \gamma S_{t-1} + \delta_1\, \Delta S_{t-1} + \delta_2 \Delta S_{t-2} + \cdots \delta_{p-1})\, \Delta S_{t-p+1} + \epsilon_t \quad (5)$$

In this more complex model, most symbols are described as having the same interpretation except for $\alpha$ is a real constant now, $\beta$ is the coefficient of the trend in time, which is also known as a drift term and lastly, the $\delta$'s are coefficients of the differences and $p$ is the lag order of the process and the last term is still the error term given by $\epsilon_t$.

Lastly, the test statistic is given by:

$$\frac{\hat{\gamma}}{SE(\widehat{\gamma})} \quad (6)$$

Where the denominator is the standard error (SE) of the regression that is fitted. In the same case as the DF test, it is expected that a value of $\gamma<0$.

### 2.2. The Hurst Exponent and Fractal Exponent

The Hurst exponent was specifically chosen due to its related features to the fractal exponent of the price time series of stocks. By analysing the Hurst exponent in conjunction with the fractal exponent, this analysis could be an integrated approach in an investor's approach to applying an applicable trading strategy.

An alternative method to test for the presence of mean reversion behaviour on a stock's price would be to analyse the diffusion speed of the stock price over a period and then comparing that result to the diffusion rate of a random walk on the same stock price. This procedure will be led to the examination of the Hurst exponent and will lead ultimately to the fractal exponent. Although there are multiple applications of the Hurst exponent in mathematics, respectively, it will look at the application of fractals and long memory processes.

Fractals can be defined as the curve of a geometric figure where each part of the geometric figure has the same statistical characteristics as the whole geometric figure. Fractals are better known to be the modelling structure of snowflakes. These patterns can be described as where similar patterns are visible and recurrent on a progressively smaller scale of the bigger or whole geometric figure. The fractal dimension measures the roughness of the surface of a geometric figure and has the following relationship to *H*:

$$H=2-D \tag{7}$$

Self-similarity is associated with fractals in this regard and one of the types of self-similarity that is also associated with applied mathematics is called statistical self-similarity. When this vision is applied to a stock price, it may be assumed that a stock price (over a long time) exhibits statistical self-similarity. It is known that any subsection of the full-time series is statistically like the full-time series.

### 2.3. Hurst Exponent and Anomalous Diffusion

One method to gain analytics into the behaviour of an asset price is to analyse the speed of the price's diffusion. Diffusion is a term used to describe the spreading out of an object, in this instance, it would be the price of a stock. This spreading out will be applied where the stock price is more concentrated in one instance than another. Applying diffusion to a stock price measures the variance of the stock price and how it depends on the difference between subsequent measurements. This measurement of diffusion can be displayed as:

$$var(\tau)=\langle|x_{t+\tau}-x_t|^2\rangle \tag{8}$$

This expression is given that $\tau$ is the time interval between two measurements and $x_t$ is a generic function of the stock price, $S_t$. The log price is:

$$x_t=log\ log(S_t) \tag{9}$$

A well-known characteristic of measuring the variance of stock price returns depends significantly on the frequency in which one decides to measure it. For example, measurements of 1-min interval historical stock prices differ significantly from measuring daily intervals. The argument can be better described by (10):

$$var(\tau)\propto\tau \tag{10}$$

(11) shows if stock prices follow a geometric random walk, which is not always the scenario, the variance of the stock price, would vary linearly with the lag of $\tau$ and the returns would also be normally distributed. However, in the case where there are small deviations from a scenario where a pure random walk is undertaken, which is the case, the variance for a given lag is often as a result of $\tau$ not being proportional to anymore. This scenario would require an anomalous exponent instead, given by:

$$var(\tau)\propto\tau^{2H} \tag{11}$$

The parameter *H* is the Hurst exponent, which has the characteristics of being both mean-reverting and described in trending stocks as:

$$H\neq\frac{1}{2} \tag{12}$$

Lastly, the daily returns of stock prices that satisfy (18) do not have a normal distribution. In this case, those returns would rather take on fatter tails and higher peaks around the mean.

By analysing the Hurst exponent further, three different market regimes can be identified, namely:

- In the case where $H<0.5$, the time series data of stock prices are mean-reverting or stationary. Log price volatility of the applicable stock price increases at a rate that is slower when compared with normal diffusion associated with geometric Brownian motion. Lastly, in this case long-term switching between high and low values in adjacent points is known as antipersistence
- If it is established that $H>0.5$, the stock price is behaving in a trending manner and is characterised as having persistence behaviour. In simple terms, higher values will be followed by even higher values and can be referred to as having a long-term autocorrelation at the stock price
- When it is given that $H = 0.5$, this scenario represents a geometric Brownian motion.

By analysing the Hurst exponent in these three different cases, it is evident that the Hurst exponent measures the level of persistence within the time series of the stock price and can furthermore be used to identify the current market state. Inevitably at a chosen time scale when the Hurst exponent changes, this could be interpreted as a signal that a shift from mean-reverting behaviour to momentum regime and vice versa.

### 2.4. Autocorrelation on Stock Prices

The applicable autocorrelation function relating to the stock price $S_t$ is:

$$\rho(\tau)=\frac{1}{\sigma^2}E\left[\left(S_t-\underline{S}\right)\left(S_{t+r}-\underline{S}\right)\right] \tag{13}$$

Processes that include autocorrelations that decay at a slow rate is termed as a long memory process. These processes have a memory from past events where past events influence future events, such as assuming on the subject of stock prices. These long memory processes are characterised by an autocorrelation function of $\rho(\tau)$ with a power-law decay with the following representation:

$$\rho(\tau \to \infty) \propto \tau^{-\alpha} \quad (14)$$

Furthermore, the relationship between $\alpha$ and $H$ is given by:

$$\alpha = 2(1-H) \quad (15)$$

It should also be noted as H approaches 1, the decay factor reaches a state where it decreases at a slower rate since the approaches the value of 0. This is then an indication of persistent behaviour. These processes may appear randomly at first but is a process of a long memory process. Having a Hurst exponent in the following interval of $\frac{1}{2} < H < 1$, is often referred to as a fractal Brownian motion.

### 2.5. Using Variance to Estimate *H*

In order to obtain the variance dependent on $\tau$, the same process is repeated or a calculation on multiple lags is done. This will ensure a slope is extracted that shows the logarithmic relationship and inevitably the importance of using multiple lag values.

It is established that a regime can be identified within a time series of stock price data by using that Hurst exponent and different interval ranges of values for it. This analysis can give an investor an important indication and insights into a particular market and the regime thereof. Also applicable to the analysis of using the Hurst exponent, the investor can establish whether to follow a mean-reverting or momentum strategy and which strategy is more appropriate to implement.

In summary, the Hurst exponent and the value thereof also indicate whether the time series of the stock price holds a long memory process, which can be described as historical events that influence future shifts. When it is found that the Hurst exponent is not always equal to 0.5, this is a further indication that the efficient market hypothesis is unpredictable and is violated by the market. This indication of anomalies within stock prices can be in principle taken advantage of by an investor and exploited to produce efficient trading strategies. This article aims to identify these anomalies and align their returns and risk tolerance to investor objectives with regard to a trading strategy.

## 3. DATA AND METHODOLOGY

### 3.1. Data

The data used in this article span over 26 years from January 1995 to December 2020. This period was chosen to include three main turbulent market periods such as the Dotcom bubble (1995–2005), the financial crisis (2008–2009), and more recently the COVID-19 pandemic (2020). Stock indices from six different stock exchange countries were chosen for comparison (also aligned to different geographical locations and developing versus developed economies).

### 3.2. Developed Economies

- Standard and Poor's 500 is a United States-based index fund that comprises the 500 largest companies, weighted by factors such as size and liquidity, on the New York Stock Exchange (Ongom et al., 2021). Standard and Poor's 500 is widely considered to be the best representation of the US stock market
- The NIKKEI 225 (Nikkei Stock Average): A Japanese index comprising 225 large companies, weighted by factors such as price and performance of the 225 largest companies traded on the Tokyo Stock Exchange across a wide variety of sectors in the Tokyo financial markets. The NIKKEI 225 is widely considered to be the best representation of the Asian stock market (Montshioa, 2021)
- The Financial Times Actuaries 100 Index is a United Kingdom-based index fund comprising 100 blue-chip stocks that are listed on the London Stock Exchange. The London Stock Exchange is the second-largest stock exchange in Europe by market capitalisation and is commonly used as the UK and global equity benchmark (Montshioa, 2021).

### 3.3. Developing Economies

- The Johannesburg Stock Exchange All Share Index is a South African-based equity index fund comprised the top-listed companies weighted by factors such as size and liquidity in South Africa. The Johannesburg Stock Exchange is the 19th largest stock exchange and largest by market capitalisation in Africa (JSE, 2017a)
- The BOVESPA (IBOVESPA): A Brazilian equity index representing the majority of trading and market capitalisation on the Brazilian Stock Exchange (Ongom et al., 2021). This index is measured by 70 public-traded companies on the B3 (Brazil Stock Exchange and over-the-counter market). This index is a weighted measurement and is a fair representation of the South American financial markets being listed as the 13th largest stock exchange in the world
- The Russian Trading System Index (RTSI): A Russian equity index comprised 50 Russian public stocks traded on the Moscow exchange (Kuramshina,2021). This is a free-float index calculated by capitalisation-weighted measurement on a 3-month review basis.

These indices were specifically chosen to provide a representation of developed and emerging markets across different geographical locations across the globe. This diversification also aligns to the theoretical object of obtaining the viability of option derivative strategy implementation use across different dynamics (such as developing versus developed countries, different indices and different maturity and strike price levels).

### 3.4. Methodology

Put option prices for the applicable hedging strategy were calculated using the methodology described in Section 3.2 using equation (6). The fractal dimension of each index was calculated as follows:

$P_i$=*Price of index on day i*
1-day log return at day *i*:

$$r_i = \ln \ln \left( \frac{P_i}{P_{i-1}} \right) \quad (16)$$

The scaling factor, the number of days a fractal dimension should return (Joshi, 2014a):

*n=scaling factor*

$$R_{i,n} = n - day \log \log return\, on\, day\, i = \ln \ln \left( \frac{P_i}{P_{i-n}} \right) \quad (17)$$

$$N_{i,n} = scaled\, return\, on\, day\, i\, with\, scaling\ factor\, n = \frac{\sum_{i-n}^{i} \left( abs\, r_i \right)}{\left[ \frac{abs\left(R_{i,n}\right)}{n} \right]} \quad (18)$$

$$D_{i,n} = fractal\ \dim ension\, on\, day\, i\, with\, scaling\ factor\, n = \frac{\ln \ln \left( N_{i,n} \right)}{\ln \ln \left( n \right)} \quad (19)$$

## 4. RESULTS

The 26-year period chosen for this analysis was characterised by both highly turbulent periods dominated by the dot-com bubble (1995–2005), the financial crisis in 2008/9, and more recently the COVID-19 pandemic in 2020, and a period of relatively non-volatile growth as shown in Figure 1.

The performance of the indices in question varies considerably indicated by end-of-term values in Figure 1. The RTSI and the BOVESPA indices grew the strongest before the dot-com bubble (end of 2005) and were least affected by this market period. The JSE ALSI, when compared to the rest of the indices, also performed relatively well during this period of turbulent market conditions. All three of these indices are categorised as developing economy indices and were least affected by the dot-com bubble, relative to the developed economies. Post this period, the developing economy indices growth continued. Following this growth spurt, all the indices' performance were influenced by the financial crisis of 2008/9, the developing economies losing most of their performance gained between 2005 and 2008, the RTSI was most affected in terms of performance during this period.

The S&P 500 and JSE ALSI have shown robust growth since the financial crisis. They both have trebled in value since January 2009 and continue apace. The NIKKEI 225 and the FTSE 100 have not increased much relative to other indices over the full period, ending the period only 39% up and 114% up, respectively, since January 1995 relative to the other indices. The COVID-19 pandemic of 2020's influence can be noted in Figure 1 for all the indices in question, but relative to the two previous turbulent market conditions, the indices have regained their performance track. All the indices under analysis have been influenced by the named three turbulent market conditions (dot-com bubble, financial crisis, and COVID-19 pandemic) and the effects thereof, but from a pure performance measurement perspective, the developing economies have been least affected by the market conditions considering their volatile performance movement.

Figure 2a-f shows the fractal indicator applied to the various chosen indices in accordance with the index price over the full period. There are clear breaches of the 1.25 threshold: each breach represents a signal to implement the hedging strategy.

Each index on each own exhibited different behaviour regarding the number of breaches of the chosen threshold of the fractal dimension at 1.25. In Figure 3a the number of breaches for each developing economy is shown, also in Figure 3b the number of breaches for the developed economies is shown. The RTSI had the most breaches (178) over the full-time period. In comparison, the other developing indices, the JSE ALSI (115) and BOVESPA (131) reached their peaks during the turbulent market conditions between 1996–1998, 2003–2008, and 2016–2020. The number of breaches per index during these different turbulent time periods shows that more breaches are indicated by the fractal dimension, which is the first step in implementing a successful hedging strategy to limit the downside risk of the strategy. More hedging positions were implemented for the RTSI index between 1996–1998, 2004–2008 whereas the JSE ALSI and RTSI had more hedging positions between 1998–2000 and 2016–2020. Thus, there were more volatile market price movements for the latter indices and their downside risks could be hedged using a put option.

**Figure 1:** Relative performance of the six indices from January 1995 to December 2020
Source: Author compilation

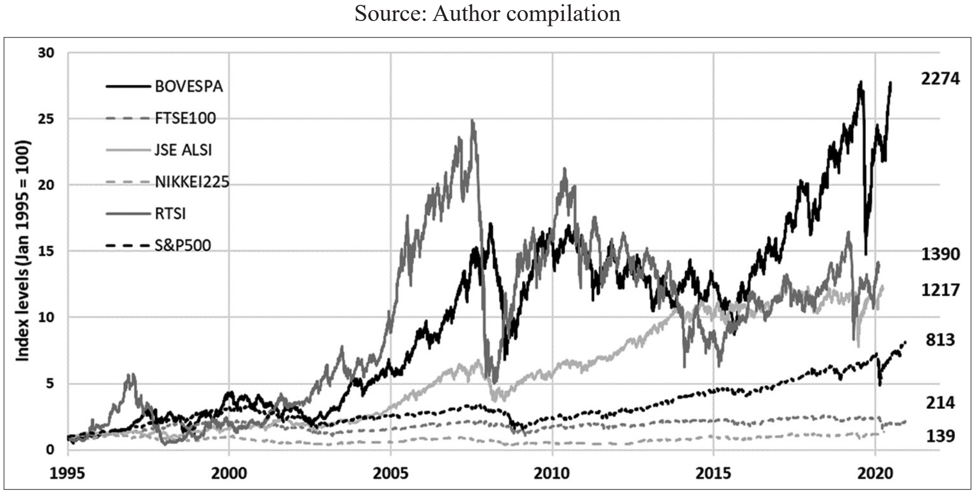
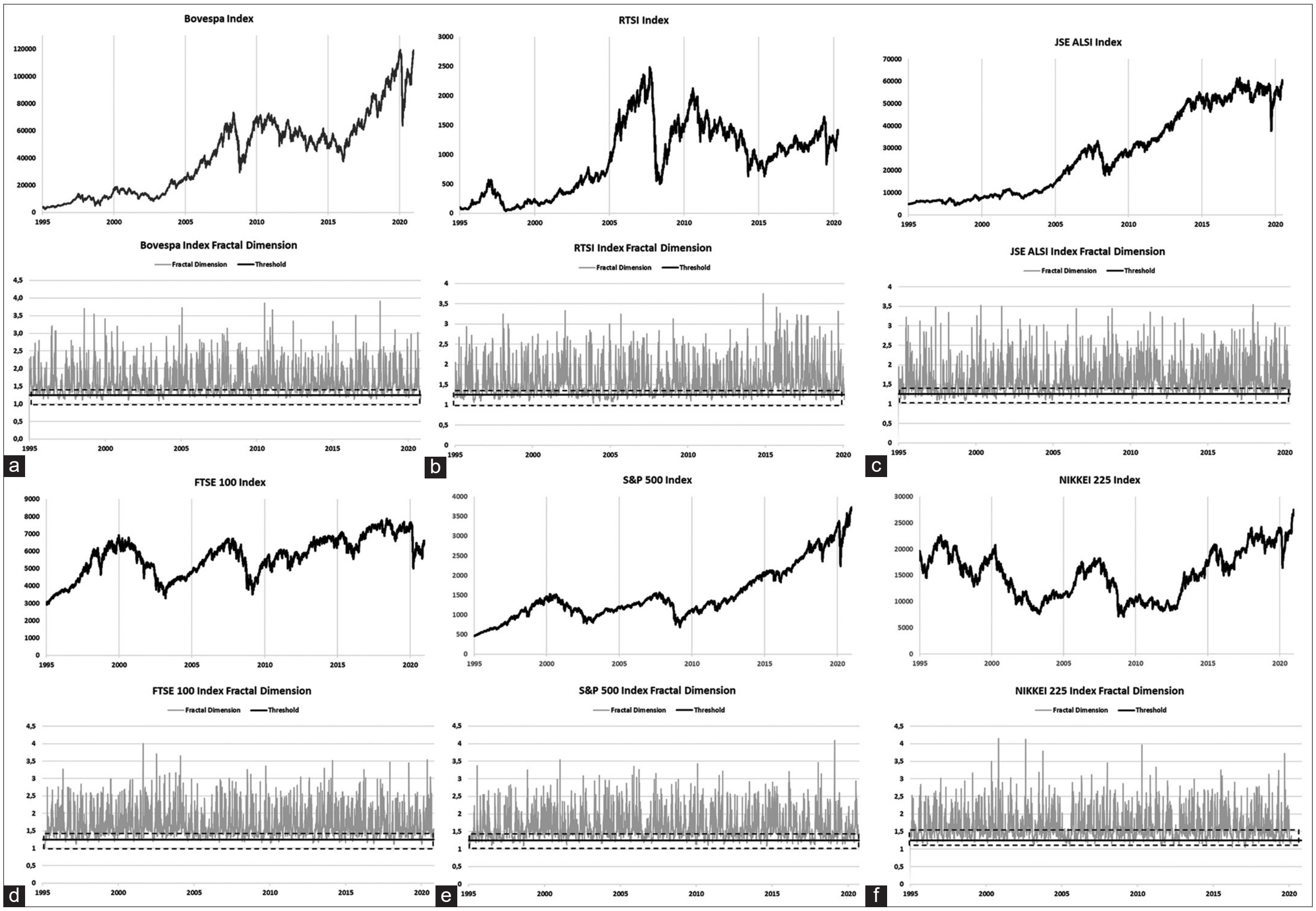


**Figure 2:** (a) Fractal dimension of the BOVESPA showing breaches. (b) Fractal dimension of the RTSI showing breaches. (c) Fractal dimension of the JSE ALSI showing breaches. (d) Fractal dimension of the FTSE 100 showing breaches. (e) Fractal dimension of the S&P 500 showing breaches. (f) Fractal dimension of the NIKKEI 225 showing breaches

Source: Author compilation




In the case of the developed economies (Figure 3b) the S&P 500 Index had the most breaches (111) over the full-time period. The FTSE 100 (76) and NIKKEI 225 (91) reached a comparatively lower number of breaches as well when compared to the developing economies, where their peaks were reached during 1995–1997, 2005–2008, and 2016–2020. Therefore, an indication of less volatile market price movements for the developed indices and less of a possibility to manage their downside risks of using a put option to hedge their risk. The number of breaches per index during these different turbulent time periods shows that fewer breaches are indicated by the fractal dimension making it difficult to implement the strategy successfully. More hedging positions were implemented for the S&P 500 index between 1995–1998, 2017–2020 whereas the FTSE 100 and NIKKEI 225 had more hedging positions between 2005 and 2008.

There are clear differences in the performance of each index. The fractal hedging strategy outperformed for the BOVESPA Index (1932%) and RTSI (3004%) while for the JSE ALSI (496%), it underperformed when compared to the other developing economies. Since the JSE ALSI showed relatively lower breaches over the full period, the assumption is made this factor also contributed to the overall strategy performance. This assumption is described by the fractal indicator not indicating more breaches for the applicable strategy to have the opportunity to limit the downside risks and limiting ultimate losses. The JSE ALSI indicated more breaches that were followed by increases in the index price where the outcome was that the index price ceased to a level higher than the strike price of the put option. The investor will therefore hold the long position in the index and only lose the premium he paid for purchasing the put option contract. The fractal dimension breaches increased slightly for the JSE ALSI in 2010–2013 whereas the BOVESPA index and RTSI showed several instances of declines and subsequent breaches in 2011–2015 and 2017, respectively. The BOVESPA Index and RTSI increased on average by 9.1% and 8%, respectively. The cumulative return gained by the BOSVESPA Index and RTSI over the full period indicated by the fractal dimension was enough to outperform their respective index, whereas the JSE ALSI did not accumulate enough returns in order to set off the return from the JSE ALSI itself. This comparison, shown in Table 1, indicates that the JSE ALSI had fewer large price movements when compared to the price increases of the BOVESPA index and RTSI.

In comparison to the developed economies, Table 1 indicates that the S&P 500 had fewer large price movements when compared

**Figure 3:** (a and b) Count of Fractal dimension breaches per year and index
Source: Author compilation

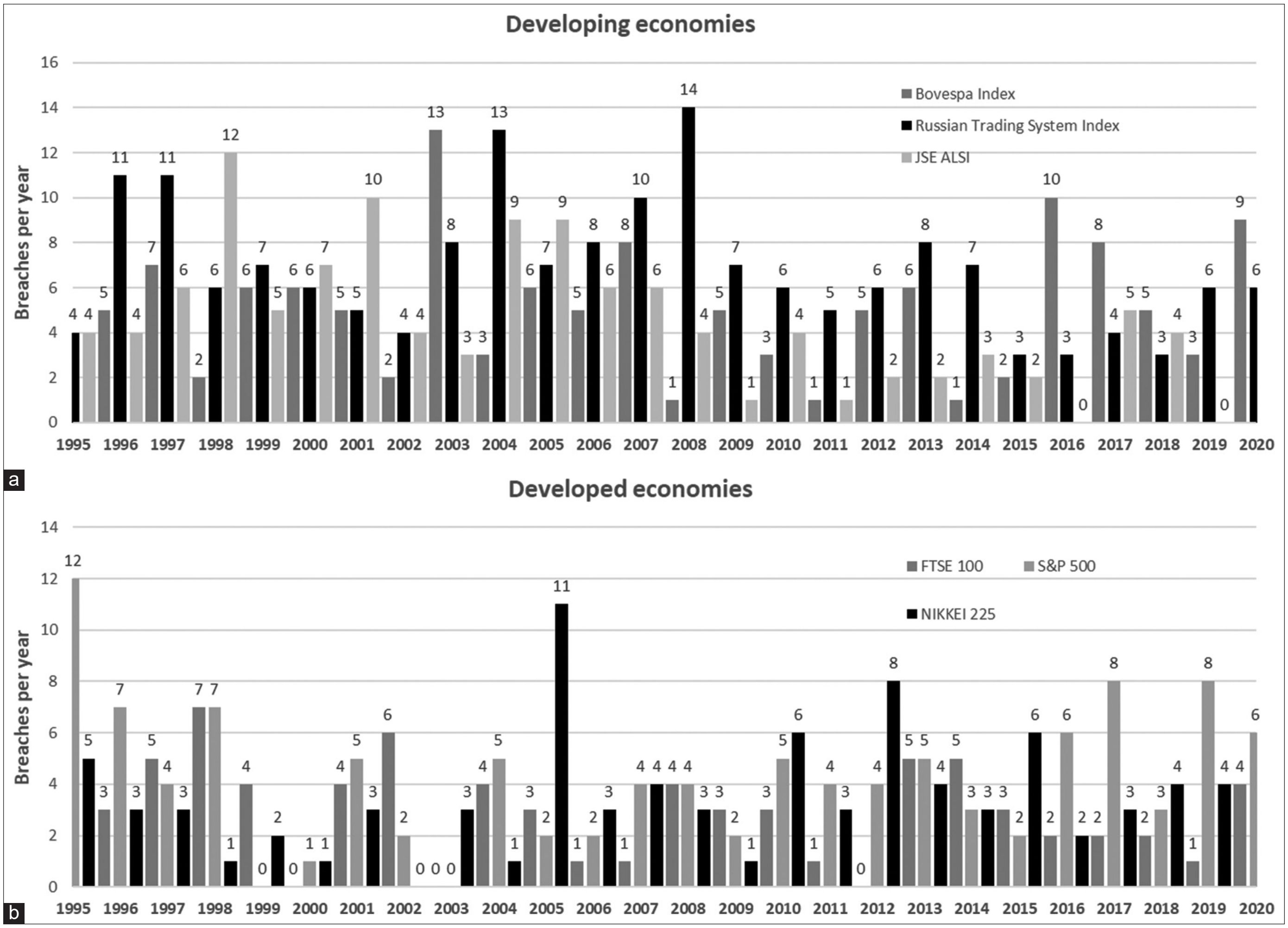
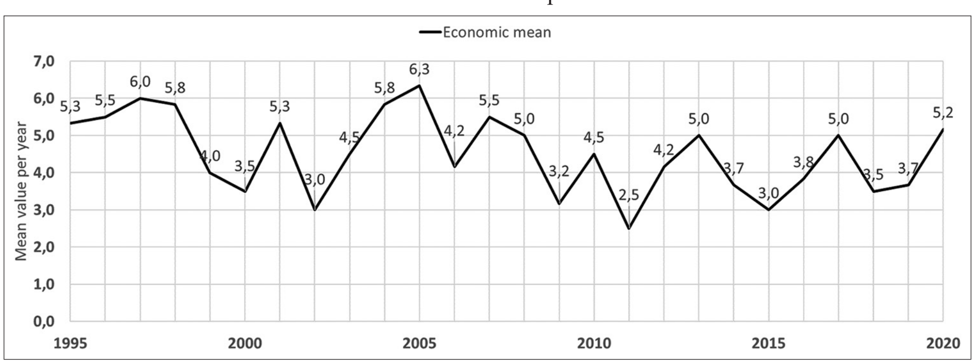



**Figure 4:** Economic mean of selected indices
Source: Author compilation




to the price increases of the FTSE 100 and NIKKEI 225. The NIKKEI 225 had 14% on average per year to ultimately increase by the largest percentage over this period. The S&P 500 and FTSE 100 increased on average by 7.2% and 5.9%, respectively. The cumulative return gained by the FTSE 100 and NIKKEI 225 was enough to outperform their respective index whereas the S&P 500 did not accrue enough daily returns from the strategy to set off the return from the S&P 500 itself.

The economic mean of the selected indices was also compared in Figure 4. The same conclusions can be drawn mentioned previously when compared to the results from Figure 3a and b. The economic mean is higher in 1997, 2005, 2008, 2013, 2017 and 2020. These relatively higher means may be influenced by the number of breaches from the RTSI in 1997–1998, the NIKKEI 225 in 2005, RTSI in 2008, NIKKEI 225 and RTSI in 2013, S&P 500 and BOVESPA in 2013, and BOVESPA and S&P 500 in 2020.

The number of breaches of the fractal indicator per index during non-turbulent and turbulent time periods (dot-com bubble, financial crisis, and COVID-19 pandemic) which indicates that more breaches are needed to limit the downside risk of holding the investment in a long position. Relative to the other years a higher number of breaches are indicated in 1997–1998, 2005, 2006, 2008, 2013, and 2020 which can be attributed to the volatile index movement of the respective turbulent market conditions.

**Figure 5:** Cumulative daily returns for each index
Source: Author compilation

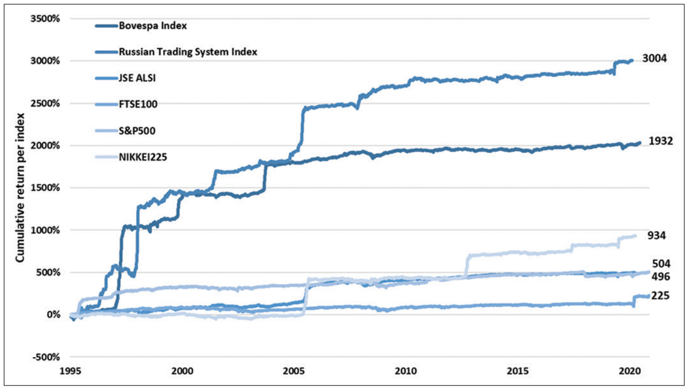
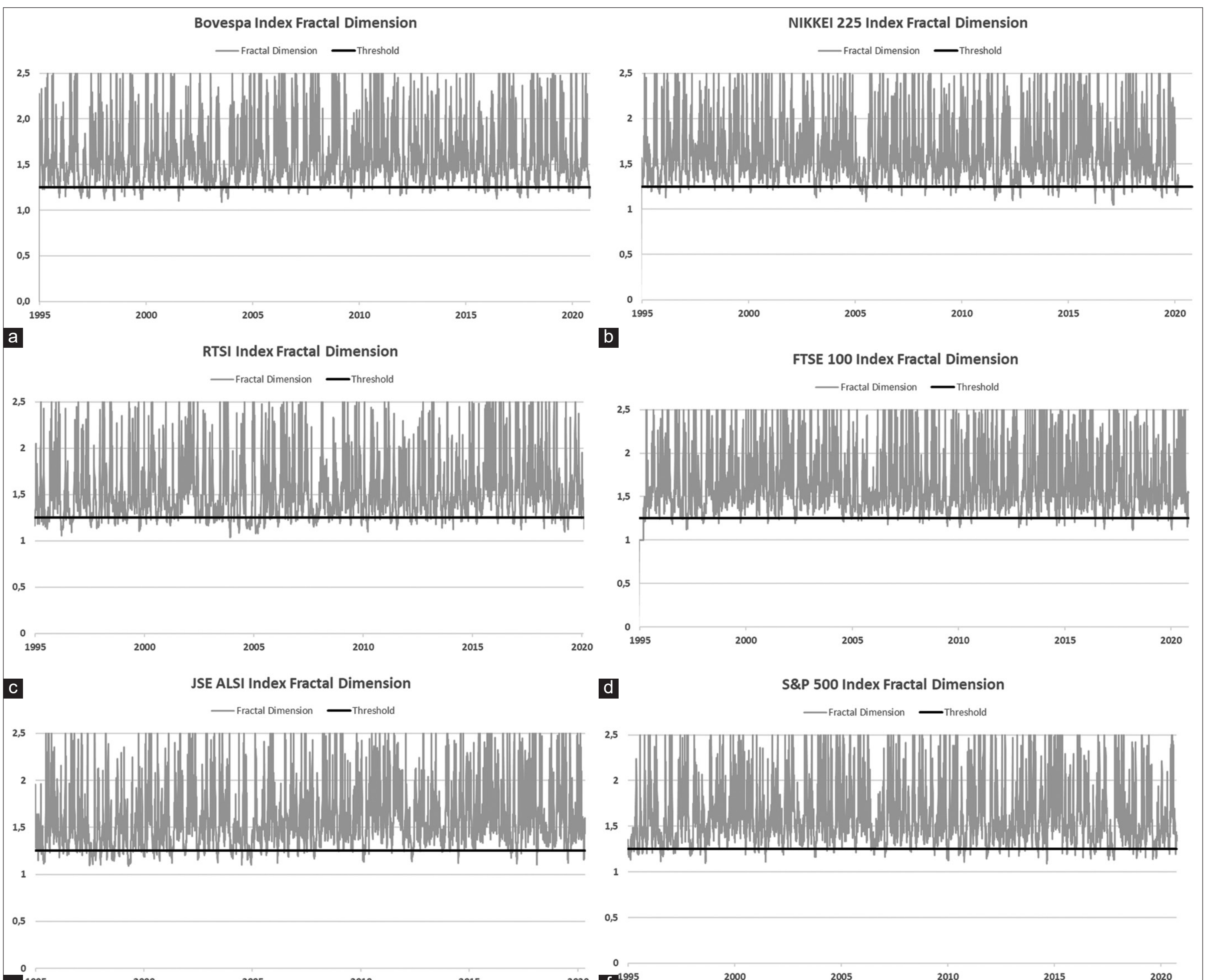



**Table 1: Cumulative daily returns**

| Cumulative returns | | | | | |
|---|---|---|---|---|---|
| BOVESPA Index | | Russian Trading System Index | | JSE ALSI | |
| Index | Hedge strategy | Index | Hedge strategy | Index | Hedge strategy |
| 1682% | 1932% | 1299% | 3005% | 1122% | 497% |
| FTSE100 | | S&P500 | | NIKKEI225 | |
| Index | Hedge strategy | Index | Hedge strategy | Index | Hedge strategy |
| 113% | 226% | 711% | 505% | 40% | 934% |

Source: Author compilation

**Figure 6:** (a and b) BOVESPA and NIKKEI 225 fractal dimensions. (c and d) RTSI and FTSE 100 Fractal dimensions. (e and f) JSE ALSI and S&P 500 Fractal dimensions
Source: Author compilation



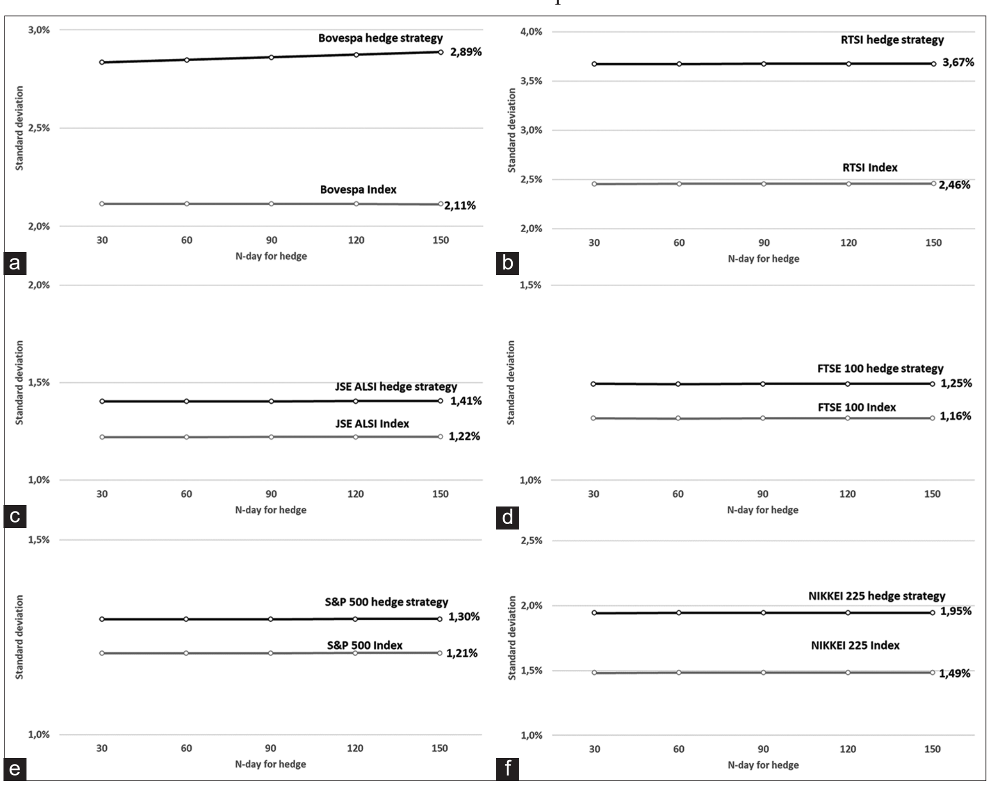
More volatile market conditions constituted a more hedged position opened during these time periods. Also, for consideration is the cumulative daily returns for each index, respectively, from Figure 5. When compared with 1998 towards the end of 2020 the cumulative daily return for the RTSI has outperformed relative to the JSE ALSI and BOVESPA Index. From 1995 towards the end of 1998 the cumulative daily return for the NIKKEI 225 has outperformed relative to the FTSE 100 and S&P 500. Afterwards, consolidation is seen for a brief period in the first half of 2005 to the end of 2013 where the NIKKEI 225 returns to a trending trajectory, with the BOVESPA and RTSI performing the best relative to the other two indices. Supporting this performance is the number of breaches indicated by Figure 6a-f for each index, respectively. The RTSI has consecutive breaches from 2009 to 2016, which is an indication of trending behaviour.

A comparison of the risk taken by each index when compared to implementing the hedging strategy on the same index with different time horizons chosen for which the put option is held, shows that risk increases as the duration of the put option increases for each index in Figures 7a-f. Also to note is that the risk takes for each taken is also higher for each index. The volatility for each index also increases over the duration of the put option and ultimately is still increasing or stays relatively level as the duration increases over the chosen duration. Comparing the results from Table 1 (cumulative returns) the BOVESPA Index and RTSI had higher cumulative returns compared to the level of risk taken per hedging strategy. The NIKKEI 225 also had the highest return for the hedging strategy; the highest levels of risk taken for implementing the hedging strategy for the developed economies. Also, the developing economies have for each index a higher level of risk taken relative to the developed economies. Relative to the other indices, the FTSE 100 exhibited lower levels of risk undertaken over different horizons and in combination with the observation made where the JSE ALSI and FTSE 100's daily cumulative return did not outperform the applied hedge strategy. This is due to the scenario where it was observed that both indices had the least number of breaches over the full period. Also contributing to this fact is their nominal growth was relatively lower than that of the other indices in question.

It is observed that when there is substantial disagreement among financial market participants the fractal dimension is less accurate in forecasting large price movements and identifying anomalies in financial markets and this scenario is in the front for market liquidity indication. The fractal dimension calculation is also a quantitative method of showcasing what the market liquidity is at a given point-in-time. By holding a long position in an index and then simultaneously with that determining what the fractal dimension is daily and then implementing a hedging strategy only when there is a breach in the chosen threshold, could limit the downside risk for the investor.

For the case where the hedging strategy applied to the BOVESPA Index, RTSI, FTSE 100, and NIKKEI 225 all outperformed their respective index performance showed by daily cumulative returns. However, in the case of the strategy applied to the S&P 500 and JSE ALSI the indices outperformed the applied hedge strategy. Deeper exploration shows that the S&P 500 and JSE ALSI had relatively small nominal growth

**Figure 7:** (a and b): BOVESPA and RTSI indices standard deviation over different hedge horizons. (c and d) JSE ALSI and FSTE 100 indices standard deviation over different hedge horizons. (e and f) S&P 500 and NIKKEI 225 indices standard deviation over different hedge horizons
Source: Author compilation



when compared to the rest of the indices. Also contributing to this anomaly is the amount of breaches the S&P 500 and JSE ALSI experienced over the full-time period. All six indices had breaches during the turbulent market conditions of the dot-com bubble, the financial crisis of 2008/2009, and more recently the COVID-19 pandemic, but where the BOVESPA Index, RTSI, FTSE 100, and NIKKEI 225 still showed several breaches thereafter, the S&P 500 and JSE ALSI's number of breaches decreased and had a relatively lower number of breaches in total. The combination of these factors in conjunction are the contributing factors to implementing the strategy in a successful manner that could be profitable to the investor according to their investment goals.

## 5. CONCLUSION AND RECOMMENDATIONS

This paper explored the implementation of a fractal dimension indicator (showing a change in liquidity) with a predetermined threshold combined with an option-based hedging strategy for six different geographical locations as well as the two different economy types. A put option contract was purchased in the event of a breach in threshold to limit the downside risk of the strategy. The fractal indicator was found to be effective when applied to four of the six tested indices in terms of cumulative returns, but also in effect increased the risk taken by the investor for all six indices. When the hedging strategy was applied to the BOVESPA Index and RTSI the daily cumulative returns were 1932% and 3004%, while if the hedging strategy was not implemented the daily cumulative returns would have decreased to 1682% and 1299%, respectively. For the developed economies when the hedging strategy was applied to the FTSE 100 and NIKKEI 225 the daily cumulative returns were 225% and 934%, for when if the hedging strategy was not applied the daily cumulative return would have only returned 113% and 40%, respectively. For further observation the same strategy was applied to both developing and developed countries. The conclusion can be made that where the outcome was similar for each economy type, both had a scenario where two out of the three economies outperformed the underlying index and had one index not outperforming the underlying index. This comparison was done to establish whether the hedging strategy had a more promising application to a developing or developed economy type. This performance measurement would have indicated if the EMH does not hold within an economy type of its respective financial market, the FMH would be used as an alternative view of market sentiment. Motivating this view is the impact of implementing a strategy to take advantage of the proposed strategy.

Furthermore, the threshold is chosen, 1.25, has been empirically determined. For fractal dimensions less than this value, herd behaviour rises and liquidity evaporates. This empirically determined threshold of 1.25 could be optimised for each economy type. In addition, for this paper, indices were selected for applying the hedging strategy. Future work could investigate whether the strategy should be applied to a single selected stock or different asset classes (commodities, financials) and compare the performance of each chosen scenario separately.

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