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Risk-free Yields, Risk Aversion, and Volatility

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ABSTRACT

The classic approach to risk analysis is rooted in the belief that risk aversion is constant, determined by constant preferences. It is becoming clear that this proposition is no longer acceptable. Risk aversion can change over short time, between sovereign countries, and on different financial and capital assets. Secondly volatility of asset prices is itself variable, and can be apprehended like the VIX volatility index which is so popular. Risk-free (RF) yields are affected by this variability in aversion and volatility, contrary to what is commonly envisioned, and contrary to what intuition suggests. This paper assumes complete markets, and simulates 14 values for the volatility, and 25 values for the coefficient of relative risk aversion (CRRA), and it measures the impact of these changes on the RF yield. One conclusion is that the CRRA is indeterminate, and is therefore consistent with the many different estimates in the literature. Another conclusion is that, by setting the volatility to 17.5%, roughly the average stock market volatility over a long period, there is evidence that the range of the implied risk premiums correspond to the range in the empirical literature.

Keywords: Risk Aversion, Volatility, Risk-free Yields, Consumption Strata, Simulation
JEL Classifications: D81, G12, G13

1. INTRODUCTION

There is a widespread belief that the risk-free (RF) interest rate is not affected by risk aversion. For example, a classic textbook in financial management predicts that, with higher risk aversion, the Security Market Line, which links systematic risk, or beta risk, to expected return, rotates leftward around the RF rate (Brigham and Houston, 2013). The result is a higher expected return for a given systematic risk or a lower affordable systematic risk for a given expected return. In brief, with a higher risk aversion one selects the portfolio with the higher return or with the lower systematic risk. One purpose of this paper is to argue otherwise: The RF rate changes with a given change in the coefficient of relative risk aversion (CRRA), after allowance is made for volatility.

The literature about the relation between the RF interest rate and risk aversion is scant. Weil (1989) is one early indirect exception. Initially Weil's intent is to solve the equity premium puzzle by using the Epstein–Zin utility function (Kreps and Porteus, 1978; Epstein and Zin, 1989, 1991; Weil; 1989). This function is particularly interesting because relative risk aversion differs, or is disentangled, from the elasticity of intertemporal substitution. Weil's findings support the hypothesis that the actual RF interest rate is much lower than the theoretical one when the expected utility paradigm is employed with such an Epstein–Zin utility function. He describes this as a puzzle and he dubs it as an interest rate puzzle, which happens to be in the paper's title. However in his Tables 1 and 2, page 413, the figures on the diagonal indicate cases where the risk aversion parameter is equal to the reciprocal of the elasticity of intertemporal substitution. There will be extensive use of these figures.

In this paper the relation between the RF interest rate and risk aversion is deepened further. The basic scaffold is still that of the maximization of expected utility. However the utility function is kept simple and the two parameters discussed above, the relative risk aversion and the elasticity of intertemporal substitution, are taken to be the reciprocal of each other. With such fundamentals the relative risk aversion is denoted by the Greek symbol γ. The utility function utilized is usually christened as being isoelastic:

$$U(C) = \frac{C^{1-\gamma} - 1}{1-\gamma} \quad \text{with } \gamma > 0$$

Table 1: Indeterminacy of the CRRA (CRRA or γ)

RF rate=rf %	CRRA γ	Volatility σ %	Premium $\gamma\sigma^2$%
3.6250	0.6	35.0	7.3500
3.7345	1.0	27.5	7.4250
3.7033	1.2	25.0	7.5000
3.9905	1.6	22.5	8.1000
3.9349	2.0	20.0	8.0000
3.6131	2.4	17.5	7.3500
3.9045	2.6	17.5	7.9625
3.7376	3.4	15.0	7.6500
3.9475	3.6	15.0	8.1000
3.6371	4.8	12.5	7.5000
3.7794	5.0	12.5	7.8125
3.782491	2.563636	20.0	7.704545

CRRA: Coefficient of relative risk aversion

Table 2: The CRRA (γ) and the RF yield for a volatility of 17.5%

CRRA γ	RF rate %	Risk premium % 0.175*0.175*γ*100
0.2	0.3054	0.6125
0.4	0.6107	1.2250
0.6	0.9156	1.8375
0.8	1.2200	2.4500
1.0	1.5235	3.0625
1.2	1.8262	3.6750
1.4	2.1277	4.2875
1.6	2.4280	4.9000
1.8	2.7268	5.5125
2.0	3.0241	6.1250
2.2	3.3196	6.7375
2.4	3.6131	7.3500
2.5	3.7591	7.6563
2.6	3.9045	7.9625
2.8	4.1936	8.5750
3.0	4.4804	9.1875
3.2	4.7646	9.8000
3.4	5.0461	10.4125
3.6	5.3248	11.0250
3.8	5.6006	11.6375
4.0	5.8733	12.2500
4.2	6.1428	12.4950
4.4	6.4091	13.4750
4.6	6.6720	14.0875
4.8	6.9314	14.7000
5.0	7.1872	15.3125

CRRA: Coefficient of relative risk aversion, RF: Risk-free

Where, C is the argument of the utility function U, and it stands frequently for consumption. This function collapses to log utility when γ is exactly equal to +1.

Although the utility function is simple my paper argues and finds evidence that the higher the risk aversion the higher the RF rate as Weil has indirectly found without paying too much attention to this regularity. On top of that my paper shows that a given RF rate is consistent with more than one CRRA. Therefore this coefficient is actually undetermined. This leads to the general conclusion that the CRRA is a random variable, which explains why, in the literature, the estimated CRRA varies along a wide range, going from 1, i.e., log utility, to 5. The fact that the CRRA is random has gained prominence lately (Azar, 2010; 2011; 2017; Dacy and Hasanov, 2011; Yoon and Byun, 2012; Gándelman and Hernández-Murillo, 2015; Conine et al., 2016). This corresponds to the frequent terminology on business channels which claim that the "appetite for risk may and does change in the stock market on an intra-day and a daily basis." Appetite for risk is the other way to label risk aversion. Unfortunately one of the implications of a random CRRA is that the utility function is also random, meaning that preferences are not constant, and this is a something that few economists would willingly approve, but which is highly productive although still provocative.

2. REVIEW

To my knowledge, Azar (2010; 2011) is the first one to have modeled risk aversion (CRRA) as a random variable. However the thrust of his endeavor in these two studies is to assess the cost of eliminating foreign exchange risk (Azar, 2010), and the cost of eliminating systematic risk (Azar, 2011). He reaches the conclusion that the RF rate is inversely related to the CRRA. This is intuitively the case because the higher the risk aversion is the lower the risk appetite, or technically, the lower the risk tolerance. As the risky asset is sold in favor of the RF asset the latter is bid up and the risk-less interest rate falls. In Azar (2010) a risk aversion of 0.5 produces the highest RF yield (5.0719%). A risk aversion of 12 produces the lowest RF yield (0.5681%). In Azar (2011) a risk aversion of 0.5 produces a risk-less interest rate of 6.47% while a CRRA of 6 produces a rate of −0.13%, which is not statistically significantly different from zero.

Weil (1989), and as mentioned above, provides for estimates of the RF interest rate for various levels of the CRRA and for two different assumptions about the rate of time preference. I choose the second table because I believe that a rate of time preference of 0.98 is more reasonable than a rate of 0.95. The CRRA and the RF rate vary in the same direction. Risk neutrality, i.e., a γ of zero, corresponds to a RF interest rate of 2.04%, while a γ of +1, i.e., log utility, corresponds to a rate of 3.75%. A CRRA of 5 corresponds to a rate of 9.56%. With a RF rate of 3.75%, it seems that log utility is not rejected.

Dacy and Hasanov (2011) construct a series of a synthetic mutual fund, which, they argue, is more representative of the market return that is usually utilized empirically. They estimate a variant of the consumption capital asset pricing model for six periods, and for both consumption of nondurable goods (ND) and consumption of ND goods plus services (NDS). With the ND series their results range between 0.81 and 4.95, while with the NDS series the range is tighter between 1.59 and 2.97. Among the 28 estimates of the CRRA they report only one is <+1, and is statistically insignificantly different than zero. This means that log utility, which has a CRRA of +1, is not supported.

Yoon and Byun (2012) study risk aversion in three international financial markets, and estimate the relation between risk aversion and three option markets, one for the Standard and Poor (S and P) 500, one for the KOSPI 200 index, and one for the Nikkei 225. Their conclusion is that investors in the S and P 500 have a higher risk aversion and higher volatility risk premiums. They find that risk aversion vary inter-country within a range of 1.56 and 10.64. Gandelman and Hernández-Murillo (2015) study also inter-country measures of the CRRA. They use happiness data from the 2006 Gallup World Poll to estimate how fast the marginal utility of income declines as income increases. The CRRA of most countries hover around +1, and the authors feel confident in recommending the use of log utility for economic analysis and simulations. Conine et al. (2016) estimate the variation of risk aversion over time using a three-moment asset pricing model that includes skewness characteristics. They succeed in proving that risk aversion varies widely across time as volatility changes. They also find that "a CRRA of roughly 1.0 is associated with a market return of roughly 7%, while a CRRA of 2.0 (roughly the average for the entire sample period) is associated with an average market return of 11.5%."

Hence a random risk aversion that varies inter-market, inter-country, across time, and for other reasons should by now be well accepted in the academic community. What is left to prove is that variations of risk aversion are linked directly to variations of the RF interest rate, and directly to variations in volatility. The point that volatility affects risk aversion is natural and intuitive. However the point that RF rates vary in tandem with risk aversion is less intuitive. The opposite is more intuitive. The first positive relation can be explained as follows: A higher CRRA means that investors are requiring a higher return for the same risk. A higher required return in a two-state pricing model means in turn that the risk-neutral probability of the higher return is itself higher. Since the RF interest rate and the risk-neutral probability vary positively then it follows that RF rates vary positively with risk aversion.

3. CONSUMPTION STRATUMS

In their seminal paper on the equity premium (Mehra and Prescott, 1985) the authors found out that the two first statistical moments of the actual equity premium require a much too elevated CRRA to be consistent with the probability density distribution function of aggregate consumption. They assumed a representative agent and argued that this agent consumes all of the economy's aggregate consumption.

Since Markowitz (1952) we know that the variance of a portfolio of random variables is lower than the variance of a single variable as long as the correlation coefficient between these variables is strictly less than perfect. Nonetheless the expected return on the portfolio is a weighted-average of individual expected returns. This is where I part with the above two authors. As a start consider that the volatility of a well-diversified portfolio of stocks in the stock market to be approximately 17.5%. Assume also that the consumption of one individual is independent from the consumption of another one. Here there are two forces at play. Altruistic behavior and caring about the fate of the other or of the fellow citizen imply a negative association between individual consumption levels. However, keeping up with the Jones's would predict a positive association. Since the problem is not settled it is reasonable to assume independence between individual consumption levels. With the assumed stock volatility of 17.5% it is easy to show that 24 consumers which face the same uncertainty in the stock market would face the same aggregate risk of aggregate consumption which is 3.6%:

$$\text{Variance} = 0.036^2 \cong 0.035722^2 = \left(\frac{24*0.175*0.175}{24*24}\right)$$

The figure of 24 stocks can be considered to be the number of income strata in the US economy. While the consumption level varies I assume that the volatility in each stratum is the same. As will be shown later the average growth rate of consumption is irrelevant to the analysis.

If the volatility of consumption is assumed to be 35%, 94 strata of independent consumers need to be assumed to get an aggregate volatility of 3.6%:

$$\text{Variance} = 0.036^2 \cong 0.0360997^2 = \left(\frac{94*0.35*0.35}{94*94}\right)$$

If the population of the United States is approaching 320 million, having around 94 strata, or less, of independent consumers is not unreasonable. The important thing is that the aggregate volatility of consumption of 3.6% is consistent with a huge number of stratum volatilities. Moreover since market volatility is by all standards a random variable, i.e., it is volatile itself, then the needed stratums to simulate aggregate consumption volatility become all probable and acceptable. This means that an individual volatility of consumption of 35%, i.e., double the volatility of a well-diversified portfolio of stocks, is probable and achievable. The issue about volatility is crucial because volatility is one of the factors that will be varied in order to prove the indeterminacy of the CRRA. In order to clarify the above statement about strata, volume of trading can be a proxy for strata. The model hence assumes that a high volume of trading is associated with a high number of strata, and a high level of volatility.

4. THE SETUP

The setup is an economy with two states of nature and two independent securities. The market is therefore complete. The annual consumption stream for the up-state is e^{σ}, where σ is the volatility of consumption of the individual consumer. The consumption stream for the down-state is the inverse of that of the up-state, i.e., $e^{-\sigma}$. The current consumption stream, or the price of the stock, is normalized to be unity. From now on consumption streams and the price of the stock are irreversibly defined to be the same. The consumer is interested with expected utility. Her function stands as:

$$E(U) = (\pi((C\exp(\sigma))^{1-\gamma}-1) + (1-\pi)((C\exp(-\sigma))^{1-\gamma}-1))/(1-\gamma)\exp(rf)$$

Assuming a certainty-equivalent above and since C is normalized to be +1, then this equation collapses to:

$$0 = \pi(\exp(\sigma))^{1-\gamma} + (1-\pi)\,(\exp(-\sigma))^{1-\gamma}-1 \qquad (1)$$

I solve for π, the probability of consumption in the upstate by numerical methods and by assuming in advance a figure for σ, the volatility of consumption growth. As argued above this volatility can take any value depending on the number of stratums one is willing to model. In my simulation the volatility σ can take the following 14 values: 0.036, 0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.2, 0.225, 0.25, 0.275, 0.3, 0.325, and 0.35. The first value, 0.036, is the volatility of the actual growth of aggregate consumption. In the simulations the coefficients of relative risk aversion take the following 25 values: Beginning by 0.2 and by increment of 0.2 up to the level of 5. These values are comparable to those deemed acceptable and reasonable by the academic community. Finally the RF interest rate is solved simultaneously by using the classic risk-neutral probability equation:

$\pi = (\exp(rf) - \exp(-\sigma))/(\exp(\sigma) - \exp(-\sigma))$

Once the probability π is known, the RF interest rate rf can be solved for. In total the sample size is 350. Therefore 350 numerical simulations, by the SOLVER command in Excel, are calculated by setting equation (1) to equal zero, and next by retrieving the riskless rate of return. What is noteworthy is that the expected returns on the two assets, the stock and its derivative, are not needed in the simulations, as I am attempting a risk-neutral analysis.

5. THE ANALYSIS

The simulation results are tabulated in the appendix and this transparency is intentional to provide a latitude for original research based on the same subject. Some features of the results are noteworthy. The higher the underlying volatility is the higher the return on the RF asset. Moreover the higher the CRRA the higher the RF return. A volatility of 3.6%, equal to average consumption growth with one stratum, does not produce reasonable results. The highest RF return for a CRRA of 5 and a volatility of 3.6% is 0.3231%. Obviously this is far away from the actual RF asset return of a quarterly 1%, or an annual of 4%. The choice of a CRRA of 5 as the maximum allowed CRRA follows the results in Azar (2006) who found a market CRRA of 4.5. However more recently findings in Azar and Karaguezian-Haddad (2014) produce a market CRRA of 2.649 when a volatility of 17% and a RF return of 3.9% are assumed. This depends on the fact that the risky asset follows a Gaussian distribution. When a volatility of 17% and a CRRA of 2.649 are assumed in this paper's simulation the resulting estimate of the RF rate of return is 3.7558%, very close to the assumed RF return in Azar and Karaguezian-Haddad (2014) of 3.9%. Hence this paper produces results that are in line with the literature. However with the assumed volatility is slightly higher at 17.5% instead of 17%, the obtained RF return is still 3.7591% when the CRRA is 2.5. Therefore with the actual data, and with 24 consumption strata, the predicted CRRA is between 2.5 and 2.7. If one assumes that the risk premium ifs equal to $\gamma\sigma^2$ then the range for this risk premium is between 7.65% and 8.27%, figures which are highly reasonable.

This discussion may be misleading in that there is one CRRA that is implied from the data. In fact the CRRA is indeterminate, depending upon the assumed volatility. Table 2 summarizes the results where a RF return between 3.6% and 4% is assumed. There are 11 possibilities for the implied CRRA, ranging from 0.6 to the maximum CRRA which is 5. Since the main argument of this paper is that volatility is itself volatile, then these 11 possibilities are all probable. One cannot know the extent of risk aversion without having a measure of the number of consumption strata, and consequently the underlying volatility. This volatility ranges between 35% and 12.5%, and the implied risk premiums, from the formula $\gamma\sigma^2$, range between 7.35% and 8.1%, figures which are reasonable. As a conclusion more than one measure of relative risk aversion is consistent with market data. The implied consumption strata are between 12 and 95. The implied volume of trading index is from a minimum of 100 to a maximum of 800. If indeed the actual volume of trading can change at most eightfold then our results are robust.

Holding volatility at its average value of 17.5% the implied RF yields vary with the CRRA. Hence the RF yields are themselves indeterminate (Table 2). These yields vary from a low of 0.3054% with a CRRA of 0.2 to a high of 7.1872% with a CRRA of 5. The equity premiums range between 0.6125% for a CRRA of 0.2 to 15.3125% for a CRRA of 5. These minimum and maximum figures are all reasonable, and have occurred in the market place. In what concerns the risk premiums they vary form a low of 0.6125% to a high of 15.3125%. Incidentally Azar (2015) finds a range for the equity premium between −0.15% and 15.1967% for arithmetic averages and between −1.2836% and 16.6124% for geometric averages.

Table 3 presents the estimates of regressions explaining the RF rate, divided by 100, with the CRRA and the volatility. Three regressions are tabulated. The two regressions with a high goodness of fit, higher than 99%, involve a log-log specification for the first one, and a quadratic regression equation with an interactive term for the second one. All coefficients are highly significant statistically and the actual P values shown in parentheses are all lower than 0.0001. However the first regression is the one we will discuss. If one sets the RF rate at 0.038, which is the actual average over a long period, then a relation between the CRRA and the volatility obtains. By solving for the CRRA this relation is:

$CRRA = 6.01453 - 22.56522\ \sigma$ with $\sigma < 0.26654$ for the CRRA to be positive.

Of course a volatility which is higher than 26.654% can and does occur in all likelihood but the RF rate of return will not be any longer at 3.8%. This limit only means that the CRRA is zero when the RF return is 3.8% and when the volatility is 26.654%. This feature of inconsistency in regression analysis is known to be frequent when predictions are made from outside the sample ranges.

Another estimate of the cut-off volatility is when a CRRA equal to zero is replaced in the quadratic regression equation with an interactive dummy. This cut-off is for a volatility of 39.1995%. In our sample the probability of a volatility ≤39.1995% is obviously +1, since the maximum simulated volatility is 35%. This means that the RF rate of return is positive under all simulated scenarios. If the volatility is taken to be 3.6%, which is the minimum volatility simulated, the implied RF rate is

Table 3: Regressions of the RF rate of return on the CRRA and the underlying volatility (σ)

independent variables	RF	Log(RF)	RF
Constant	−0.079945 (0.0000)	3.827748 (0.0000)	0.018645 (0.0000)
Log (CRRA)	-	0.973499 (0.0000)	-
Log(σ)	-	1.963201 (0.0000)	-
CRRA	0.019610 (0.0000)	-	−0.005206 (0.0000)
$CRRA^2$	-	-	−0.000870 (0.0000)
Σ	0.442504 (0.0000)	-	−0.345272 (0.0000)
$σ^2$	-	-	1.006768 (0.0000)
CRRA*σ	-	-	0.155817 (0.0000)
Adjusted R^2	0.820131	0.999652	0.994224

CRRA: Coefficient of relative risk aversion, RF: Risk-free

3.048% at a zero CRRA. This is highly reasonable compared to actual happenings.

The log-log regression in Table 3 has a very high goodness of fit, with an adjusted R^2 of 0.999652. The results show that a 1% increase in the CRRA increases the RF rate by about 1%. However this coefficient, i.e., 0.973499, while economically very close to +1, is statistically significantly different from +1 with a t-statistic of −13.860. The impact of volatility is stronger: A 1% increase in volatility, i.e., from 12% to 12.12%, increases the RF rate by 1.9632%. Therefore a 100% increase in the volatility, or a doubling of the volatility, quadruples the RF interest rate. As an example a change of the volatility between 10% and 20%, which is reasonable in the stock market, changes the RF rate by around 4 times, from a value of 0.4912% to a value of 1.9868, when the CRRA is assumed to be +1.

6. CONCLUSION

The main thrust of this paper is to start from the hypothesis of the existence of a variable degree of risk aversion. Such a hypothesis is found to be more fruitful than expected. I append to this hypothesis the concept that there are many independent consumer strata, and that the market is complete. Some of the results are as follows. There is a positive relation between risk aversion and the RF rate. To this is added another positive relation between the RF rate and volatility. Moreover the risk aversion coefficient is indeterminate, i.e., consistent with many RF returns between 3.6% and 4%. This indeterminacy explains why there are so many different point estimates of the CRRA. If one assumes a benchmark stock volatility of 17.5% then the implied equity risk premium is estimated to be between 0.6125% and 15.3125%, a range which is consistent with unconditional interval estimates of this premium (Azar, 2015). Finally, and from the simulated data, regressions are run to explain the RF rate by two variables, the CRRA and volatility. From one of these regressions an equation linking the CRRA and volatility is derived under the assumption that the RF rate is 3.8%. Since the paper is based upon a variable risk aversion, the analysis here will find a favorable echo with business practitioners who comment oftentimes on business channels that market changes in risk aversion have occurred, or are likely to have occurred.

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APPENDIX

Appendix Table 1: The data

CRRA	RF rate	Volatility
0.2	0.013	0.036
0.2	0.025	0.05
0.2	0.056	0.075
0.2	0.0999	0.1
0.2	0.156	0.125
0.2	0.2246	0.15
0.2	0.3054	0.175
0.2	0.3986	0.2
0.2	0.504	0.225
0.2	0.6216	0.25
0.2	0.7513	0.275
0.2	0.8931	0.3
0.2	1.0467	0.325
0.2	1.2122	0.35
0.4	0.0259	0.036
0.4	0.05	0.05
0.4	0.112	0.075
0.4	0.1998	0.1
0.4	0.312	0.125
0.4	0.449	0.15
0.4	0.6107	0.175
0.4	0.7969	0.2
0.4	1.0076	0.225
0.4	1.2425	0.25
0.4	1.5016	0.275
0.4	1.7845	0.3
0.4	2.0912	0.325
0.4	2.4215	0.35
0.6	0.0389	0.036
0.6	0.075	0.05
0.6	0.1686	0.075
0.6	0.2997	0.1
0.6	0.4679	0.125
0.6	0.6733	0.15
0.6	0.9156	0.175
0.6	1.1946	0.2
0.6	1.5101	0.225
0.6	1.8619	0.25
0.6	2.2496	0.275
0.6	2.6729	0.3
0.6	3.1315	0.325
0.6	3.625	0.35
0.8	0.0518	0.036
0.8	0.1	0.05
0.8	0.2248	0.075
0.8	0.3995	0.1
0.8	0.6237	0.125
0.8	0.8972	0.15
0.8	1.22	0.175
0.8	1.5913	0.2
0.8	2.0111	0.225
0.8	2.4789	0.25
0.8	2.9942	0.275
0.8	3.5566	0.3
0.8	4.1654	0.325
0.8	4.8201	0.35
1.0	0.0648	0.036
1.0	0.1249	0.05
1.0	0.281	0.075
1.0	0.4992	0.1
1.0	0.7792	0.125
1.0	1.1208	0.15

(*Contd...*)

Appendix Table 1: (*Continued*)

CRRA	RF rate	Volatility
1.0	1.5235	0.175
1.0	1.9868	0.2
1.0	2.5102	0.225
1.0	3.093	0.25
1.0	3.7345	0.275
1.0	4.4341	0.3
1.0	5.1908	0.325
1.0	6.0039	0.35
1.2	0.0777	0.036
1.2	0.1499	0.05
1.2	0.3371	0.075
1.2	0.5988	0.1
1.2	0.9345	0.125
1.2	1.3439	0.15
1.2	1.8262	0.175
1.2	2.3807	0.2
1.2	3.0067	0.225
1.2	3.7033	0.25
1.2	4.4694	0.275
1.2	5.304	0.3
1.2	6.2059	0.325
1.2	7.1738	0.35
1.4	0.0907	0.036
1.4	0.1749	0.05
1.4	0.3932	0.075
1.4	0.6983	0.1
1.4	1.0896	0.125
1.4	1.5663	0.15
1.4	2.1277	0.175
1.4	2.7728	0.2
1.4	3.5003	0.225
1.4	4.3091	0.25
1.4	5.1978	0.275
1.4	6.1648	0.3
1.4	7.2086	0.325
1.4	8.3274	0.35
1.6	0.1036	0.036
1.6	0.1999	0.05
1.6	0.4493	0.075
1.6	0.7976	0.1
1.6	1.2442	0.125
1.6	1.7881	0.15
1.6	2.428	0.175
1.6	3.1627	0.2
1.6	3.9905	0.225
1.6	4.9098	0.25
1.6	5.9187	0.275
1.6	7.0152	0.3
1.6	8.1972	0.325
1.6	9.4624	0.35
1.8	0.1166	0.036
1.8	0.2248	0.05
1.8	0.5052	0.075
1.8	0.8968	0.1
1.8	1.3985	0.125
1.8	2.0091	0.15
1.8	2.7269	0.175
1.8	3.5502	0.2
1.8	4.4769	0.225
1.8	5.5047	0.25
1.8	6.6313	0.275
1.8	7.854	0.3

(*Contd...*)

Appendix Table 1: (*Continued*)

CRRA	RF rate	Volatility
1.8	9.17	0.325
1.8	10.576	0.35
2.0	0.1295	0.036
2.0	0.2497	0.05
2.0	0.5612	0.075
2.0	0.9959	0.1
2.0	1.5524	0.125
2.0	2.2292	0.15
2.0	3.0241	0.175
2.0	3.9349	0.2
2.0	4.9589	0.225
2.0	6.0932	0.25
2.0	7.3346	0.275
2.0	8.6799	0.3
2.0	10.125	0.325
2.0	11.668	0.35
2.2	0.1425	0.036
2.2	0.2747	0.05
2.2	0.6171	0.075
2.2	1.0947	0.1
2.2	1.7058	0.125
2.2	2.4483	0.15
2.2	3.3196	0.175
2.2	4.3166	0.2
2.2	5.4362	0.225
2.2	6.6745	0.25
2.2	8.0278	0.275
2.2	9.4917	0.3
2.2	11.07	0.325
2.2	12.734	0.35
2.4	0.1554	0.036
2.4	0.2996	0.05
2.4	0.6729	0.075
2.4	1.1933	0.1
2.4	1.8587	0.125
2.4	2.6663	0.15
2.4	3.6131	0.175
2.4	4.6951	0.2
2.4	5.9084	0.225
2.4	7.2483	0.25
2.4	8.71	0.275
2.4	10.289	0.3
2.4	11.978	0.325
2.4	13.774	0.35
2.6	0.1683	0.036
2.6	0.3245	0.05
2.6	0.7286	0.075
2.6	1.2917	0.1
2.6	2.011	0.125
2.6	2.8833	0.15
2.6	3.9045	0.175
2.6	5.0701	0.2
2.6	6.3751	0.225
2.6	7.8138	0.25
2.6	9.3807	0.275
2.6	11.069	0.3
2.6	12.873	0.325
2.6	14.786	0.35
2.8	0.1813	0.036
2.8	0.3494	0.05
2.8	0.7843	0.075
2.8	1.3898	0.1
2.8	2.1627	0.125
2.8	3.0989	0.15
2.8	4.1936	0.175

(*Contd...*)

Appendix Table 1: (*Continued*)

CRRA	RF rate	Volatility
2.8	5.4413	0.2
2.8	6.8359	0.225
2.8	8.3707	0.25
2.8	10.039	0.275
2.8	11.833	0.3
2.8	13.746	0.325
2.8	15.77	0.35
3.0	0.1942	0.036
3.0	0.3742	0.05
3.0	0.8398	0.075
3.0	1.4876	0.1
3.0	2.3137	0.125
3.0	3.3133	0.15
3.0	4.4804	0.175
3.0	5.8085	0.2
3.0	7.2905	0.225
3.0	8.9185	0.25
3.0	10.684	0.275
3.0	12.579	0.3
3	14.595	0.325
3	16.723	0.35
3.2	0.2071	0.036
3.2	0.3991	0.05
3.2	0.8953	0.075
3.2	1.5852	0.1
3.2	2.4641	0.125
3.2	3.5262	0.15
3.2	4.7646	0.175
3.2	6.1715	0.2
3.2	7.7386	0.225
3.2	9.4566	0.25
3.2	11.3161	0.275
3.2	13.3074	0.3
3.2	15.4205	0.325
3.2	17.6456	0.35
3.4	0.22	0.036
3.4	0.4239	0.05
3.4	0.9507	0.075
3.4	1.6825	0.1
3.4	2.6137	0.125
3.4	3.7376	0.15
3.4	5.0461	0.175
3.4	6.5301	0.2
3.4	8.1798	0.225
3.4	9.9847	0.25
3.4	11.934	0.275
3.4	14.0165	0.3
3.4	16.221	0.325
3.4	18.5365	0.35
3.6	0.2329	0.036
3.6	0.4487	0.05
3.6	1.0059	0.075
3.6	1.7794	0.1
3.6	2.7626	0.125
3.6	3.9475	0.15
3.6	5.3248	0.175
3.6	6.8841	0.2
3.6	8.614	0.225
3.6	10.5025	0.25
3.6	12.5374	0.275
3.6	14.7062	0.3
3.6	16.9962	0.325
3.6	19.3954	0.35
3.8	0.2458	0.036
3.8	0.4735	0.05

(*Contd...*)

Appendix Table 1: (*Continued*)

CRRA	RF rate	Volatility
3.8	1.0611	0.075
3.8	1.876	0.1
3.8	2.9106	0.125
3.8	4.1557	0.15
3.8	5.6006	0.175
3.8	7.2332	0.2
3.8	9.0407	0.225
3.8	11.0096	0.25
3.8	13.126	0.275
3.8	15.3761	0.3
3.8	17.7459	0.325
3.8	20.2222	0.35
4.0	0.2587	0.036
4.0	0.4982	0.05
4.0	1.1161	0.075
4.0	1.9722	0.1
4.0	3.0577	0.125
4.0	4.3622	0.15
4.0	5.8733	0.175
4.0	7.5774	0.2
4.0	9.4599	0.225
4.0	11.5058	0.25
4.0	13.6996	0.275
4.0	16.0259	0.3
4.0	18.4698	0.325
4.0	21.01657	0.35
4.2	0.2716	0.036
4.2	0.523	0.05
4.2	1.171	0.075
4.2	2.068	0.1
4.2	3.204	0.125
4.2	4.567	0.15
4.2	6.1428	0.175
4.2	7.9164	0.2
4.2	9.8713	0.225
4.2	11.9908	0.25
4.2	14.2578	0.275
4.2	16.6555	0.3
4.2	19.1677	0.325
4.2	21.7787	0.35
4.4	0.2845	0.036
4.4	0.5477	0.05
4.4	1.2258	0.075
4.4	2.1634	0.1
4.4	3.3493	0.125
4.4	4.7698	0.15
4.4	6.4091	0.175
4.4	8.2501	0.2

(*Contd...*)

Appendix Table 1: (*Continued*)

CRRA	RF rate	Volatility
4.4	10.2747	0.225
4.4	12.4646	0.25
4.4	14.8	0.275
4.4	17.2647	0.3
4.4	19.8397	0.325
4.4	22.5087	0.35
4.6	0.2974	0.036
4.6	0.5724	0.05
4.6	1.2805	0.075
4.6	2.2585	0.1
4.6	3.4937	0.125
4.6	4.9707	0.15
4.6	6.672	0.175
4.6	8.5784	0.2
4.6	10.67	0.225
4.6	12.9264	0.25
4.6	15.3274	0.275
4.6	17.8534	0.3
4.6	20.4858	0.325
4.6	23.207	0.35
4.8	0.3102	0.036
4.8	0.597	0.05
4.8	1.335	0.075
4.8	2.3531	0.1
4.8	3.6371	0.125
4.8	5.1697	0.15
4.8	6.9314	0.175
4.8	8.9011	0.2
4.8	11.0569	0.225
4.8	13.3767	0.25
4.8	15.8385	0.275
4.8	18.4216	0.3
4.8	21.1062	0.325
4.8	23.874	0.35
5.0	0.3231	0.036
5.0	0.6216	0.05
5.0	1.3894	0.075
5.0	2.4473	0.1
5.0	3.7794	0.125
5.0	5.3666	0.15
5.0	7.1872	0.175
5.0	9.2182	0.2
5.0	11.4355	0.225
5.0	13.8152	0.25
5.0	16.3339	0.275
5.0	18.9695	0.3
5.0	21.7013	0.325
5.0	24.5102	0.35

CRRA: Coefficient of relative risk aversion