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Bayesian Approach for Indonesia Inflation Forecasting

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ABSTRACT

This paper presents a Bayesian approach to find the Bayesian model for the point forecast of ARMA model under normal-gamma prior assumption with quadratic loss function in the form of mathematical expression. The conditional posterior predictive density is obtained from the combination of the posterior under normal-gamma prior with the conditional predictive density. The marginal conditional posterior predictive density is obtained by integrating the conditional posterior predictive density, whereas the point forecast is derived from the marginal conditional posterior predictive density. Furthermore, the forecasting model is applied to inflation data and compare to traditional method. The results show that the Bayesian forecasting is better than the traditional forecasting.

Keywords: ARMA Model, Bayes Theorem, Inflation, Normal-gamma Prior
JEL Classifications: C13, C15, C22

1. INTRODUCTION

Bayes theorem calculates the posterior distribution as proportion to the product of a prior distribution and the likelihood function. The prior distribution is a probability model describing the knowledge about the parameters before observing the currently by the available data. Main idea of Bayesian forecasting is the predictive distribution of the future given the fast data follows directly from the joint probabilistic model. The predictive distribution is derived from the sampling predictive density, weighted by the posterior distribution (Bijak, 2010).

This paper is refers to Amry and Baharum (2015) discussing the problem of Bayesian forecasting for ARMA model under Jeffrey' prior. Other papers related to this research are Amry (2016), Fan and Yao (2008), Kleibergen and Hoek (1996), and Uturbey (2006) also discussed the Bayesian analysis for ARMA model. This paper focuses to find the mathematical expression of the Bayes estimator for the point forecast of ARMA model under normal-gamma prior assumption with quadratic loss function and to compare to traditional method.

2. MATERIALS AND METHODS

The materials in this paper are some theories in mathematics and statistics such as the ARMA model, Bayes theorem, repeated integration, and the univariate student's t-distribution and inflation data. The method is study of literatures by applying the Bayesian analysis under normal-gamma prior assumption.

ARMA (p, q) model (Liu, 1995) is defined by:

$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + e_t \tag{1}$$

Where $\{e_t\}$ is sequence of *i i d* normal random variables with $e_t \sim N(0,\tau^{-1})$, $\tau>0$ and unknown, φ_i and θ_j are parameters.

The Bayes theorem (Ramachandran and Tsokos, 2009) stated as:

$$p(y|x)\ \alpha\ p(x|y)\ p_y\ (y) \tag{2}$$

Where $p(y|x)$ is posterior distribution, $p(x|y)$ is likelihood function and $p_y(y)$ is prior distribution.

A random quantity, X, is said to have a student distribution on n degrees of freedom with mode μ and scale parameter τ>0 if it has the probability density function (Pole et al., 1994):

$$p(X) = \frac{\Gamma\left(\frac{n+1}{2}\right) n^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)(\pi\tau)\frac{1}{2}} \left[n + \frac{(x-\mu)^2}{\tau} \right]^{-\frac{n+1}{2}} \tag{3}$$

The mean is $E(X)=\mu$ and the variance is $Var(X)=\dfrac{n\tau}{n-2}$, if $n>2$

In the Bayesian approach the point forecast determined based the Bayes estimator. Rever to Bain & Engelhardt (2006), if is an estimator of, then a quadratic loss function is any real-valued function:

$$L(\hat{\theta};\theta)=\left(\hat{\theta}-\theta\right)^2 \quad (4)$$

For the quadratic loss function, the Bayes Estimator is the mean of the posterior distribution (DeGroot, 2004).

3. RESULTS

The k−step−ahead point forecast of y_{n+k}, is defined by:

$$\tilde{y}(K)=E(y_{n+k}\mid S_n^*) \quad (5)$$

Where $S_n^*=(y_1,y_2,\ldots,y_{n+k-1})$

$$e_t=y_t-\sum_{i=1}^{p}\phi_i y_{t-i}-\sum_{j=1}^{p}\phi_i e_{t-j}$$

Based on the equation (1) can be obtained residuals:

$$e_t=y_t-\sum_{i=1}^{p}\varphi_i y_{t-i}-\sum_{j=1}^{q}\theta_j e_{t-j} \quad (6)$$

By conditioning the first p observations and letting $e_p=e_{p-1}=\ldots=e_r=0$, where r = min(0, p+1−q), one may approximate by Box & Jenkins, the likelihood function for parameters $\Psi=(\phi_1, \phi_2,\ldots,\phi_p, \theta_1, \theta_2,\ldots, \theta_q)$ and τ based is:

$$L(\psi,\tau\mid S_n^*)\alpha\propto\tau^{\frac{(n+k-1)-p}{2}}$$
$$\exp\left\{\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1}\left(y_t-\sum_{i=1}^{p}\phi_i y_{t-i}-\sum_{j=1}^{q}\theta_j e_{t-j}\right)^2\right]\right\} \quad (7)$$

The equation (7) can be expressed as:

$$L(\psi,\tau\mid S_n^*)\propto\tau^{\frac{(n+k-1)-p}{2}}$$
$$\exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1}y_t^2-2\psi^T\sum_{t=p+1}^{n+k-1}y_t B_{t-1}+\sum_{t=p+1}^{n+k-1}\left(\psi^T B_{t-1}\right)^2\right]\right\} \quad (8)$$

Where

$B_t=(y_t, y_{t-1},\ldots y_{t+1-p}, e_t, e_{t-1},\ldots,e_{t+1-q})$

By letting

$$U=\begin{bmatrix} y_p & y_{p+1} & \cdots & y_{n+k-2}\\ y_{p-1} & y_p & \cdots & y_{n+k-3}\\ \vdots & \vdots & \vdots & \vdots\\ y_1 & y_2 & \cdots & y_{n+k-1-p}\\ e_p & e_{p+1} & \cdots & e_{n+k-2}\\ e_{p-1} & e_p & \cdots & e_{n+k-3}\\ \vdots & \vdots & \vdots & \vdots\\ e_{p+1-q} & e_{p+2-q} & \cdots & e_{n+k-1-q}\end{bmatrix}, X_0=\begin{bmatrix} y_{p+1}\\ y_{p+2}\\ \vdots\\ y_{n+k-1}\end{bmatrix}$$

Where

$$e_t=y_t-\sum_{i=1}^{p}\tilde{\phi}_i y_{t-i}-\sum_{j=1}^{q}\tilde{\theta}_j\tilde{e}_{t-j}, t=p+1,p+2,\ldots n,\tilde{\phi}_i$$

And are maximum likelihood estimator of . The values of e_t, $e_{t-1},\ldots,e_{t-q}$ can be obtained via:

$$e_t=y_t-\tilde{\psi}^T B_{t-1} \quad (9)$$

Where

$$\tilde{\psi}=(\tilde{\phi}_1,\tilde{\phi}_2,\ldots\tilde{\phi}_p,\tilde{\theta}_1,\tilde{\theta}_2,\ldots\tilde{\theta}_q)$$

From the likelihood function in equation (8) can be obtained:

$$\sum_{t=p+1}^{n+k-1}y_t B_{t-1}=y_{p+1}B_p+y_{p+2}B_{p+1}+\ldots y_{n+k}B_{n+k-2}$$
$$=y_{p+1}(y_p,y_{p-1},\ldots,y_1,e_p,e_{p-1},\ldots,e_{p+1-q})+$$
$$y_{p+2}(y_{p+1},y_p,\ldots,y_2,e_{p+1},e_p,\ldots,e_{p+2-q})+\ldots+$$
$$y_{n+k}(y_{n+k-2},y_{n+k},\ldots,y_{n-k-1-p},e_{n+k-2},\ldots,e_{n+k-1-q})$$
$$=U^T X_0$$

$$\sum_{t=p+1}^{n+k-1}\left(\Psi^T B_{t-1}\right)^2=\left(\Psi^T B_p\right)^2+\left(\Psi^T B_{p+1}\right)^2+\ldots+\left(\Psi^T B_{n+k-2}\right)^2$$
$$\left(\begin{matrix}(\varphi_1,\varphi_2,\ldots\varphi_p,\theta_1,\theta_2,\ldots,\theta_q)^T\\(y_p,y_{p-1},\ldots,y_1,e_p,e_{p-1},\ldots,e_{p+1-q})\end{matrix}\right)^2+$$
$$\left(\begin{matrix}(\varphi_1,\varphi_2,\ldots\varphi_p,\theta_1,\theta_2,\ldots,\theta_q)^T\\(y_{p+2},y_{p+1},\ldots,y_2,e_{p+2},e_{p+1},\ldots,e_{p+2-q})\end{matrix}\right)^2+\ldots+$$
$$\left(\begin{matrix}(\varphi_1,\varphi_2,\ldots\varphi_p,\theta_1,\theta_2,\ldots,\theta_q)^T\\(y_{n+k-2},y_{n+k-3},\ldots,y_{n-k-1-p},e_{n+k-2},e_{n+k-3},\ldots,e_{n+k-1-q})\end{matrix}\right)^2$$
$$=\Psi^T(UU^T)\Psi$$

Such that the likelihood function in equation (8) can be expressed as:

$$L\left(\Psi,\tau|S_n^*\right) \propto e^{\frac{(n+k-1)-p}{2}}$$
$$\exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1} y_t^2 - 2\Psi^T(U^T X_0) + \Psi^T(UU^T)\Psi\right]\right\}$$

$$\propto e^{\frac{(n+k-1)-p}{2}} \exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1} y_t^2 - 2\Psi^T V + \Psi^T W\Psi\right]\right\} \quad (10)$$

Where $U^T X_0 = V$ and $UU^T = W$

3.1. Posterior Distribution

According to Broemeling and Shaarawy's suggestion (1988), the normal-gamma prior of parameters Ψ and τ is:

$$\xi(\Psi,\tau)=\xi_1(\Psi|\tau).\xi_2(\tau)$$
$$=\left(\frac{\tau Q}{2\pi}\right)^{\frac{p}{2}} \exp\left\{-\frac{\tau}{2}\left[(\Psi-\mu)^T Q(\Psi-\mu)\right]\right\}$$
$$x\frac{\beta^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}\exp(-\beta\tau)$$
$$\propto(\tau)^{\frac{p}{2}+\alpha-1}\exp\left\{-\frac{\tau}{2}\left[(\Psi-\mu)^T Q(\Psi-\mu)\right]-\beta\tau\right\} \quad (11)$$
$$\propto\tau^{\frac{p+2\alpha}{2}-1}\exp\left\{-\frac{\tau}{2}\left[(\Psi-\mu)^T Q(\Psi-\mu)+2\beta\right]\right\}$$
$$\propto\tau^{\frac{p+2\alpha}{2}-1}$$
$$\exp\left\{-\frac{\tau}{2}\left[\Psi^T Q\Psi-\Psi^T Q\mu-\mu^T Q\Psi+\mu^T Q\mu+2\beta\right]\right\}$$

Where $\xi_1 \sim N(\mu,(\tau Q)^{-1})$, $\xi_1 \sim GAM(\alpha,\beta)$, Q is a positive definite matrix of the order $(p+q)$, α and β are parameters. By applying the Bayes theorem to equation (10) and (11), the posterior distribution of ψ and τ^{-n} is:

$$\pi\left(\Psi,\tau^{-1}|S_n^*\right) \propto \tau^{\frac{1}{2}(n+k-1-p)}$$
$$\exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1} y_t^2 - 2\Psi^T V + \Psi^T W\Psi\right]\right\}$$

$$\tau^{\frac{p+2\alpha}{2}-1} \exp\left\{-\frac{\tau}{2}\left[\Psi^T Q\Psi-\Psi^T Q\mu-\mu^T Q\Psi+\mu^T Q\mu+2\beta\right]\right\}$$

$$\propto \tau^{\frac{n+k-1-p+2\alpha+p}{2}-1}$$
$$\exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1} \begin{matrix} y_t^2 - 2\Psi^T V + \Psi^T W\Psi + \Psi^T Q\Psi - \\ \Psi^T Q\mu - \mu^T Q\Psi + \mu^T Q\mu + 2\beta \end{matrix}\right]\right\}$$

$$\propto \tau \frac{n+k-1-p+2\alpha+p}{2}-1$$
$$\exp\left\{-\frac{\tau}{2}\left[\begin{matrix}\Psi^T(W+Q)\Psi-\Psi^T V-\Psi^T Q\mu- \\ \Psi^T V-\mu^T Q\Psi+\sum_{t=p+1}^{n+k-1} y_t^2+\mu^T Q\mu+2\beta\end{matrix}\right]\right\}$$

$$\propto \tau^{\frac{n+k-1-p+2\alpha+p}{2}-1}$$
$$\exp\left\{-\frac{\tau}{2}\left[\begin{matrix}\Psi^T(W+Q)\Psi-\Psi^T(V+Q\mu)- \\ V^T\Psi-(Q\mu)^T\Psi+\sum_{t=p+1}^{n+k-1} y_t^2+\mu^T Q\mu+2\beta\end{matrix}\right]\right\}$$

$$\propto \tau^{\frac{n+k-1-p+2\alpha+p}{2}-1}$$
$$\exp\left\{-\frac{\tau}{2}\left[\begin{matrix}\Psi^T(W+Q)\Psi-\Psi^T(V+Q\mu)- \\ (V+Q\mu)^T\Psi+\sum_{t=p+1}^{n+k-1} y_t^2+\mu^T Q\mu+2\beta\end{matrix}\right]\right\}$$

$$\therefore\ \pi\left(\psi,\tau^{-1}|S_n^*\right) \propto \tau^{\frac{n+k-1-p+2\alpha+p}{2}-1}$$
$$\exp\left\{-\frac{\tau}{2}\left[\Psi^T P\Psi-\Psi^T(V+Q\mu)-(V+Q\mu)^T\Psi+K\right]\right\} \quad (12)$$

Where $W+Q=P$ *and* $\sum_{t=p+1}^{n+k-1} y_t^2+\mu^T Q\mu+2\beta=K$

3.2. Conditional Posterior Predictive Density

Based on $e_t = y_t - \sum_{i=1}^{p}\varphi_i y_{t-i} - \sum_{j=1}^{q}\theta_j e_{t-j}$ with $e_t \sim N(0,\tau^{-1})$ can be obtained

$$f(e_t|S_n^*,\Psi,\tau^{-1})=\left(2\pi\tau^{-1}\right)^{-\frac{1}{2}}\exp\left\{-\frac{\tau}{2}(e_t)^2\right\}$$

If expressed in y_t:

$$f(y_t|S_n^*,\Psi,\tau^{-1})=\left(2\pi\tau^{-1}\right)^{-\frac{1}{2}}$$
$$\exp\left\{-\frac{\tau}{2}\left[y_t-\sum_{i=1}^{p}\varphi_i y_{t-i}-\sum_{j=1}^{q}\theta_j e_{t-j}\right]^2\right\} \quad (13)$$

Based on the equation (13), can be obtained the conditional predictive density of Y_{n+k}:

$$f(y_{n+k}|S_n^*,\Psi,\tau^{-1})=\left(2\pi\tau^{-1}\right)^{-\frac{1}{2}}$$
$$\exp\left\{-\frac{\tau}{2}\left[y_{n+k}-\sum_{i=1}^{p}\varphi_i y_{n+k-i}-\sum_{j=1}^{q}\theta_j e_{n+k-j}\right]^2\right\}$$

$$\propto \tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k} - \sum_{i=1}^{p}\varphi_i y_{n+k-i} - \sum_{j=1}^{q}\theta_j e_{n+k-j}\right]^2\right\}$$

$$\propto \tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k} - \left(\sum_{i=1}^{p}\varphi_i y_{n+k-i} + \sum_{j=1}^{q}\theta_j e_{n+k-j}\right)\right]^2\right\}$$

$$\propto \tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k}^2 - 2\Psi^T B_{n+k-1}\, y_{n+k} + \left(\Psi^T B_{n+k-1}\right)^2\right]\right\}$$

$$\propto \tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k}^2 + \Psi^T R\Psi - 2\Psi^T B_{n+k-1}\, y_{n+k} \right]\right\} \quad (14)$$

Where $B_{n+k-1} = (y_{n+k-1}, y_{n+k-2}, \ldots, y_{n+k-p}, e_{n+k-1}, \ldots, e_{n+k-2}, \ldots, e_{n+k-q})$ and $R = B_{n+k-1} \otimes B_{n+k-1}^T$

Based on the equation (12) and (14) can be obtained the conditional posterior predictive density of Y_{n+k}:

$$f_p(y_{n+k} \mid S_n^*, \Psi, \tau^{-1}) \propto \pi\left(\Psi, \tau^{-1} \mid S_n^*\right) f(y_{n+k} \mid S_n^*, \Psi, \tau^{-1})$$

$$\propto \tau^{\frac{(n+k-1-p+2\alpha)+p}{2}-1} \exp\left\{-\frac{\tau}{2}\begin{bmatrix}\Psi^T P\Psi - \Psi^T(V+Q\mu) - \\ (V+Q\mu)^T\Psi + K\end{bmatrix}\right\}$$

$$\tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k}^2 + \Psi^T R\Psi - 2\Psi^T B_{n+k-1}\, y_{n+k}\right]\right\}$$

$$\propto \tau^{\frac{(n+k-1-p+2\alpha)+p}{2}-1} \exp\left\{-\frac{\tau}{2}\begin{bmatrix}\Psi^T P\Psi - \Psi^T(V+Q\mu) - \\ (V+Q\mu)^T\Psi + K\end{bmatrix}\right\}$$

$$\tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\begin{bmatrix} y_{n+k}^2 + \Psi^T R\Psi - \Psi^T \\ B_{n+k-1}y_{n+k} - B_{n+k-1}^T y_{n+k}\,\Psi\end{bmatrix}\right\}$$

$$\propto \tau^{\frac{(n+k-1-p+2\alpha)+p+1}{2}-1}$$

$$\exp\left\{-\frac{\tau}{2}\begin{bmatrix}\Psi^T P\Psi - \Psi^T(V+Q\mu) \\ -(V+Q\mu)^T\Psi + K + y_{n+k}^2 + \Psi^T R\Psi \\ -\Psi^T B_{n+k-1}\, y_{n+k} - B_{n+k-1}^T\, y_{n+k}\,\Psi\end{bmatrix}\right\}$$

$$\propto \tau^{\frac{(n+k-1-p+2\alpha)+p+1}{2}-1}$$

$$\exp\left\{-\frac{\tau}{2}\begin{bmatrix}\Psi^T P\Psi - \Psi^T(V+Q\mu) - \\ (V+Q\mu)^T\Psi + K + y_{n+k}^2 + \Psi^T R\Psi \\ -\Psi^T B_{n+k-1}\, y_{n+k} - B_{n+k-1}^T\, y_{n+k}\,\Psi\end{bmatrix}\right\}$$

The conditional posterior predictive density of Y_{n+k} using norma gamma prior is:

$$\therefore f_p\left(y_{n+k} \mid S_n^*, \Psi, \tau^{-1}\right) \propto \tau^{\frac{(n+k-1-p+2\alpha')+p+1}{2}-1}$$

$$\exp\left\{-\frac{\tau}{2}\begin{bmatrix}\Psi^T G\Psi - \Psi^T(V+Q\mu+B_{n+k-1}\, y_{n+k}) - \\ ((V+Q\mu)^T + B_{n+k-1}^T\, y_{n+k})\Psi + y_{n+k}^2 + K\end{bmatrix}\right\} \quad (15)$$

Where $G = P + R$

3.3. Marginal Conditional Posterior Predictive

The marginal conditional posterior predictive density of Y_{n+k} can be obtained by integrating the conditional posterior predictive density in equation (15): density in equation (15):

$$f_p\left(y_{n+k} \mid S_n^*\right) = \int_0^\infty\int_{-\infty}^\infty f_p(y_{n+k} \mid S_n^*, \Psi, \tau^{-1})\, d\Psi\, d\tau$$

$$\propto \int_0^\infty\int_{-\infty}^\infty \tau^{\frac{(n+k-1-p+2\alpha)+p+1}{2}-1}$$

$$\exp\left\{-\frac{\tau}{2}\begin{bmatrix}\Psi^T G\Psi - \Psi^T(V+Q\mu+B_{n+k-1}\, y_{n+k}) - \\ ((V+Q\mu)^T + B_{n+k-1}^T\, y_{n+k})\Psi + y_{n+k}^2 + K\end{bmatrix}\right\} d\Psi\, d\tau$$

$$\propto \int_0^\infty\int_{-\infty}^\infty \tau^{\frac{(n+k-1-p+2\alpha)+p+1}{2}-1}$$

$$\left\{\begin{matrix}\exp\left\{-\frac{\tau}{2}\begin{bmatrix} y_{n+k}^2 + K - \left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)^T \\ G^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\end{bmatrix}\right\} \\ \exp\left\{-\frac{\tau}{2}\begin{bmatrix}\left(\Psi - G^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\right)^T \\ G\left(\Psi - Z^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\right)\end{bmatrix}\right\}\end{matrix}\right\} d\Psi\, d\tau$$

$$\propto \int_0^\infty \tau^{\frac{(n+k-1-p+2\alpha)p+1}{2}-1}$$

$$\exp\left\{-\frac{\tau}{2}\begin{bmatrix} y_{n+k}^2 + K - \left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)^T \\ G^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\end{bmatrix}\right\}$$

$$\left(\int_{-\infty}^\infty \exp\left\{-\frac{\tau}{2}\begin{bmatrix}\left(\Psi - G^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\right)^T \\ G\left(\Psi - G^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\right)\end{bmatrix}\right\} d\Psi\right) d\tau$$

$$\propto \tau^{\frac{(n+k-1-p+2\alpha)+p+1}{2}-1}$$

$$\exp\left\{-\frac{\tau}{2}\begin{bmatrix} y_{n+k}^2 + K - \left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)^T \\ G^{-1}\left((V+Q\mu) + B_{n+k-1}\, y_{n+k}\right)\end{bmatrix}\right\} d\tau$$

$$\propto \int_0^\infty \tau^{\frac{(n+k-1-p+2\alpha)+p+1}{2}-1} \exp\left\{-\tau\left[\frac{y_{n+k}^2+K-\left((V+Q\mu)+B_{n+k-1}\,y_{n+k}\right)^T G^{-1}\left((V+Q\mu)+B_{n+k-1}\,y_{n+k}\right)}{2}\right]\right\} d\tau \qquad (16)$$

By applying the formula of Gamma distribution from the equation (16) can be obtained:

$$f_p\left(y_{n+k} \mid S_n^*\right) \propto \left[\frac{y_{n+k}^2+K-\begin{pmatrix}(V+Q\mu)\\+B_{n+k-1}\,y_{n+k}\end{pmatrix}^T G^{-1}\left((V+Q\mu)+B_{n+k-1}\,y_{n+k}\right)}{2}\right]^{-\frac{n+k-1-2p+2\alpha+p+1}{2}}$$

$$\propto \left[K+y_{n+k}^2-\left((V+Q\mu)+B_{n+k-1}\,y_{n+k}\right)^T G^{-1}\left((V+Q\mu)+B_{n+k-1}\,y_{n+k}\right)\right]^{-\frac{(n+k-1-p+2\alpha)+1}{2}}$$

$$\propto \left[1+\left\{\frac{\left[y_{n+k}-\left(1-B_{n+k-1}^T G^{-1}B_{n+k-1}\right)^{-1}\left(B_{n+k-1}^T G^{-1}(V+Q\infty)\right)\right]^T \left(1-B_{n+k-1}^T G^{-1}B_{n+k-1}\right)\left[y_{n+k}-\left(1-B_{n+k-1}^T G^{-1}B_{n+k-1}\right)^{-1}\left(B_{n+k-1}^T G^{-1}(V+Q\infty)\right)\right]}{K-(V+Q\infty)^T G_0 (V+Q\infty)}\right\}\right]^{-\frac{(n+k-1-\square+\square\alpha)+1}{\square}}$$

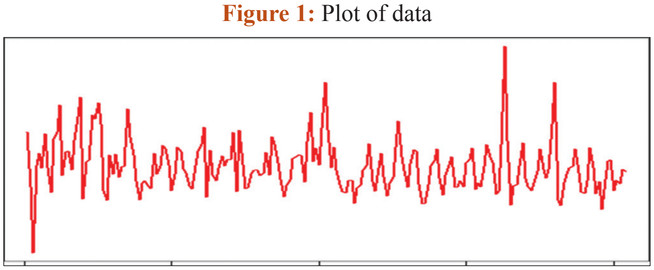

Figure 1: Plot of data

Figure 2: Plot of ACF

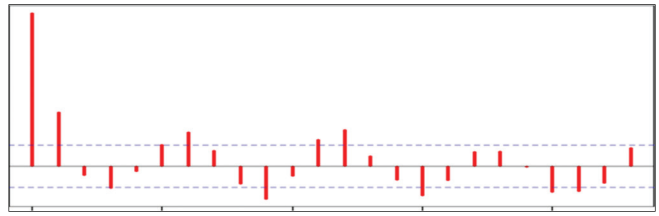

Figure 3: Plot of PACF

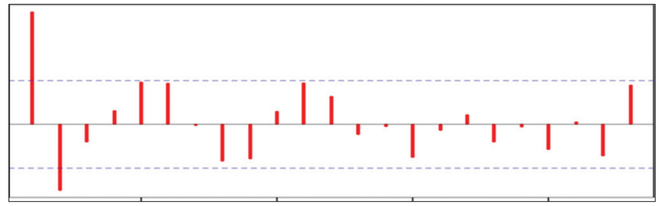

Figure 4: Plot of factual data (red), Bayesian (green) and traditional (blue)

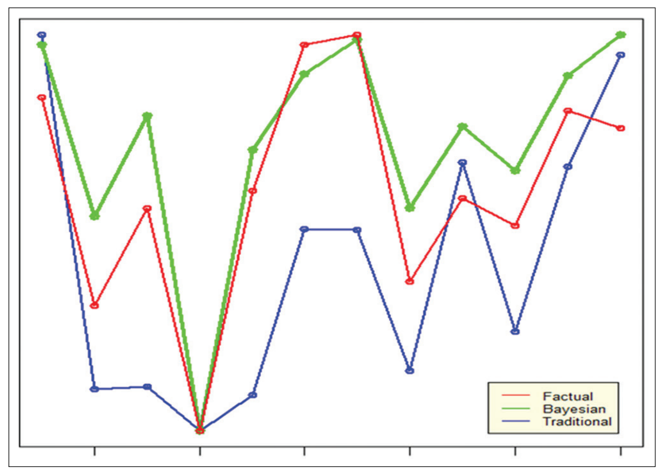

Table 1: Value of parameters and A IC

MODEL	EQ	EQ	EQ	AIC
ARMA (0,1)	-	-	0.4058	335.00
ARMA (1,0)	0.3544	-	-	340.84
ARMA (1,1)	0.0686	-	0.3528	331.79
ARMA (2,0)	0.4317	−0.2129	-	333.46
ARMA (2,1)	0.5883	−0.2712	−0.1625	334.96

Table 2: Comparison of point forecast between Bayesian with traditional forecasting

k	Factual data	Result of forecasting: EQ	
		Bayesian	Traditional
1	0.51	0.443232000	0.42096670
2	−0.09	0.006368232	0.02118361
3	0.19	0.262402900	0.02374863
4	−0.45	0.538752700	−0.02603840
5	0.24	0.175542000	0.01485990
6	0.66	0.369192000	0.20172880
7	0.69	0.455384000	0.20085360
8	−0.02	0.027196330	0.04167132
9	0.22	0.235994600	0.27691080
10	0.14	0.123299330	0.08616260
11	0.47	0.365324000	0.27197930
12	0.42	0.469548070	0.39836110

Table 3: Comparison of forecast accuracy

Forecast accuracy	Bayesian	Traditional
RMSE	0.12476883	0.2545024
MAE	0.09569079	0.1962556
MAPE	47.6807038	81.4006722
U-statistics	0.16897685	0.4092601

Table 4: Comparison of descriptive statistics

Data	Min.	Q1	Median	Mean	Q3	Max	SD
Factual 1–204	140	1.14	0.5	0.55	0.9	3.3	0.589
Factual 1–192, Bayesian 193–204	140	1.13	0.5	0.55	0.9	3.3	0.590
Factual 1–192 Trad. 193–204	140	1.12	0.4	0.54	0.9	3.3	0.588

$$\propto \left[(n+k-1-p+2\alpha) + \left\{ \frac{\left[y_{n+k} - \begin{pmatrix} 1-B_{n+k-1}^{T} \\ G^{-1}B_{n+k-1} \end{pmatrix}^{-1} \begin{pmatrix} B_{n+k-1}^{T}\,G^{-1} \\ (V+Q\mu) \end{pmatrix} \right]^{2}}{\dfrac{K-(V+Q\mu)^{T}G_{0}(V+Q\mu)}{(n+k-1-p+2\alpha)} \left(1-B_{n+k-1}^{T}\,G^{-1}B_{n+k-1}\right)} \right\} \right]^{-\frac{(n+k-1-p+2\alpha)+1}{2}}$$

Where $G_0 = G^{-1} + \left(1 - B_{n+k-1}^{T}\,G^{-1}B_{n+k-1}\right)^{-1} \left(G^{-1}RG^{-1}\right)$

The marginal conditional posterior predictive density of Y_{n+k} is a univariate student's t-distribution on $(n+k-(p+2\alpha)$ degrees of freedom with mode $\propto = \left(1 - B_{n+k-1}^{T}\,G^{-1}B_{n+k-1}\right)^{-1} \left(B_{n+k-1}^{T}\,G^{-1}(V+Q\propto)\right)$

3.4. Point Forecast

For quadratic loss function, the point forecast of Y_{n+k} is the posterior mean of the marginal conditional posterior predictive, that is:

$$E\left(Y_{n+k} \mid S_n^{*}\right) = \left(1 - B_{n+k-1}^{T}\,G^{-1}B_{n+k-1}\right)^{-1} \left(B_{n+k-1}^{T}\,G^{-1}(V+Q\propto)\right) \quad (17)$$

4. APPLICATION

The results of point forecast are applied to a set of time series data that have been identified by ARMA model using normal-gamma prior. The forecasting model is applied to the period 193–204 based on data from 1 to 192.

4.1. Data, Stationarity, Identification, and Model Selection

Data of 204 series, y, of monthly inflation in Indonesian from January 2000 to December 2016 is displayed in Figure 1. Plot of ACF in Figure 2 in the form of damped sine wave, indicates that the time series data is stationary. Plot ACF in Figure 2 is disconnected after first lag and plot of PACF in Figure 3 is disconnected after second lag, these indicate that the appropriate model for data are ARMA(0,1), ARMA(1,0), ARMA(1,1), RMA(2,0), and ARMA(2,1).

The calculation the value of parameters and the value of AIC is presented as Table 1.

The smallest AIC value in Table 1 is 331.79 on ARMA(1,1) model, it means the suitable model for the data is ARMA(1,1) model. In Y_t, its model is written:

$$Y_t = 8530606{=}0.3335Y_{t-1} + e_t \quad (18)$$

4.2. Comparison to Traditional Method

The comparison of point forecast between Bayesian forecasting in equation equation (17) with traditional forecasting in equation (18) is presented in the Table 2. Columns 2 through 4 containing the factual data, result of Bayesian forecasting, and result of traditional forecasting.

The comparison of forecast accuracy between Bayesian method and traditional method is presented in the Table 3. Rows 2 through 5 containing the RMSE, MAE, MAPE and U-Statistics.

The results show that the forecast accuracy value of the Bayesian method is smaller than the traditional method, so in this case it is concluded that the forecast accuracy for the Bayesian forecasting is better than the traditional forecasting. The comparison of descriptive statistics between the Bayesian method and the traditional method is presented in the Table 4. Columns 2 through 8 containing the minimum (Min), first quartile (Q1), median, mean, third quartile (Q3), maximum (Max), and standard deviation for 204 factual data, 192 factual data and the result of Bayesian forecasting for the 12 steps ahead, and 192 factual data and the result of traditional forecasting for the 12 steps ahead.

Plot of factual data, Bayesian and traditional forecasting for the 12 steps ahead is displayed in Figure 4, shows that the plot of factual data is more varied to the plot of Bayesian than the traditional forecasting.

5. CONCLUSION

This paper analyzes how to find out mathematical expression of the point forecast for Bayesian forecasting under normal-gamma prior. The conditional posterior predictive density is obtained by multiplying the normal-gamma prior with the conditional predictive density. The marginal conditional posterior predictive density is obtained by integrating the conditional posterior predictive density to paramaters, whereas the point forecast is derived based on the mean of marginal conditional posterior predictive density that has the univariate student's t-distribution.

The computational results show that the forecast accuracy value of Bayesian forecasting is smaller than the traditional forecasting, while the values of descriptive statistics show that the Bayesian forecasting is closer to the factual data than the traditional forecasting, it indicates that the Bayesian forecasting is better than the traditional forecasting.

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