



# Modeling and Forecasting USD/UGX Volatility through GARCH Family Models: Evidence from Gaussian, T and GED Distributions

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## ABSTRACT

Symmetric and asymmetric GARCH models-GARCH (1,1), PARCH (1,1), EGARCH (1,1), TARCH (1,1) and IGARCH (1,1) were used to examine stylized facts of daily USD/UGX return series from September 01, 2005 to August 30, 2018. Modeling and forecasting were performed based on Gaussian, Student's t and GED distribution densities to identify the best distribution for examining stylized facts about the volatility of returns. Initial tests of heteroscedasticity (ARCH-LM), autocorrelation and stationarity were carried out to establish specific data requirements before modeling. Results for conditional variance indicated the presence of significant asymmetries, volatility clustering, leptokurtic distribution, and leverage effects. Effectively, PARCH (1,1) under GED distribution provided highly significant results free from serial correlation and ARCH effects, thus revealing the asymmetric responsiveness and persistence to shocks. Forecasting was performed across distributions and assessed based on symmetric loss functions (RMSE, MAE, MAPE and Thiel's U) and information criteria (AIC, SBC and Loglikelihood). Information criteria offered preference for EGARCH (1,1) under GED distribution while symmetric loss functions provided very competitive choices with very slight precedence for GARCH (1,1) and EGARCH (1,1) under GED distribution. Following these results, we recommend PARCH (1,1) and EGARCH (1,1) for modeling and forecasting volatility with preference to GED distribution. Given the asymmetric responsiveness and persistence of conditional variance, macroeconomic fiscal adjustments in addition to stabilization of the internal political environment are advised for Uganda.

**Keywords:** Forecasting Volatility, GARCH Family Models, Probability Distribution Density, Forecast Accuracy

**JEL Classifications:** C58, C53, G17, F31

## 1. INTRODUCTION

International financial cash flows tend to be hugely affected by uncertainties due to fluctuations in key economic markets such as foreign exchange and stock markets, which results into the decline of exports and imports, which in turn affect welfare as suggested by (Twamugize et al., 2017). As such, understanding volatility has become very necessary especially now that foreign exchange markets account for the largest trade volumes and liquidities in the world.

Influential attempts aimed at modeling volatility were introduced into literature through the seminal work of Engle (1982) in which he proposed conditionalizing variance in an autoregressive heteroscedastic process by introducing the autoregressive conditional heteroscedastic (ARCH) model. This model, however, posed challenges due to huge lag specifications. This weakness motivated Bollerslev (1986) to introduce a generalized form of ARCH called the GARCH model which improved on the former by providing flexibility to the lag structure. Over time, the literature on these models has grown to capture stylized facts of financial

series such as leverage, volatility clustering, fat-tail distributions (Kipkoeh, 2014). Consequently, the following models have been developed to capture different scenarios: Exponential GARCH to unconditionally model asymmetries (Nelson, 1991); Power ARCH to model nonlinearities by specifying some restrictions (Ding et al., 1993); Threshold GARCH to analyze leverage effects based on news and Integrated GARCH to model non-stationarities. These models among others form the GARCH family models and they capture different dynamics of financial series.

The performance of the stated GARCH family models is unquestionably helpful, however, issues relating to high data frequency and increased kurtosis posit a levy distribution on return series (Mandelbrot, 1963). Modeling and forecasting of volatility should, therefore, consider the distribution of innovations that should not be assumed to be normal as opposed to conventional normality assumptions.

In recognition of these inherent gaps, this study makes attempts at introducing alternative distributions across models to capture different stylized facts of return series through careful selection of mean variances that consider the potential weakness of time series models such as the issues of autocorrelations and non-stationarities. The contributions of this paper to academic literature are twofold: The first, to the best of our knowledge we pioneer the application of GARCH family models in Uganda's exchange markets, thus augmenting Namugaya et al. (2014) who focused on stock markets. Following Coffie's (2015) contributions on distribution densities of innovations and issues relating to (a)symmetries, this paper provides extensional contributions through the introduction of PARCH and IGARCH that capture different stylized facts of returns across different distributions in both symmetric and asymmetric frameworks.

A vast literature is available and continues to grow for GARCH family models that have been used under different specifications in various disciplines to analyze volatility and stylized facts related to forex and stock markets. Musa et al. (2014) examined the performance of GARCH models using data on Naira/USD for Nigeria between the periods June 2000 and July 2011. Their findings showed that GJR GARCH provides better performances over other GARCH family models. Also, they found evidence for the existence of significant asymmetric effects. Using symmetric loss functions (MAE, RMAE, MAPE and Thiel's U), their results further showed that TGARCH provided accurate forecasts. Omari et al. (2017) used data on daily returns of KES/USD between 2003 and 2015 to investigate stylized facts about exchange rates in both symmetric and asymmetric sets of models. They specifically investigated GARCH (1,1) and GARCH-M (1,1) for symmetric models and EGARCH, GJR-GARCH (1,1) and APARCH (1,1) for the asymmetric set under different distributions. Their results indicated that APARCH, GJR-GARCH model and EGARCH models better modeled volatility with t-distribution. Coffie (2015) considers the relative performance of different GARCH family models along with different distributions across markets. His interest was specifically modeling and forecasting both asymmetric and symmetric models in Botswana and Namibia stock markets using normal, t and GED distributions. His findings revealed less

persistence of shocks and asymmetry of news in both markets and the existence of reverse volatility with models with fatter tails providing better performance. In contrast, Abdullah et al. (2017) adopts a similar strategy but only for a single market by analyzing the volatility of the Bangladesh taka against the USD with a series of daily returns running from January 2008 to April 2015. By adopting multiple mean equations to overcome diagnostics problems in GARCH, APARCH, EGARCH, TGARCH and IGARCH models, their findings revealed that student's t-distribution provided better performance over normal distribution.

GARCH family models are not limited to forex markets, but also extend to stock markets in which they are used to model stylized facts similar to those of forex markets. With a similar specific focus on Kenya, Maqsood et al. (2017) delved into the analysis of GARCH family models to model and forecast volatility by employing daily returns for the Nairobi Securities Exchange using the NSE 20 share index. His findings revealed the persistence of volatility and clustering effect, leverage effect and asymmetric response to external shocks. It was on such a basis that he concluded that NSE is an inefficient market exhibiting stylized facts of financial markets. Ahmed and Suliman (2011) undertook similar motivations to model GARCH family models by applying daily returns of the Khartoum stock exchange (KSE) from January 2006 to November 2010. By considering both symmetric and asymmetric models, their empirical results revealed that conditional variance is highly persistent (explosive process) and provides evidence on the existence of risk premium for the KSE index return series which supports the positive correlation hypothesis between volatility and the expected stock returns. Besides, they found that asymmetric models provide a better fit than the symmetric models, confirming the existence of leverage effect.

Alternative modern approaches have been advanced to model volatility. This includes the use of neural networks and ARIMA that predate GARCH family models. For instance, Ou and Wang (2011) aimed at modeling and predicting financial volatility, but on a Gaussian probabilistic process based on GARCH, EGARCH and GJR-GARCH models by training different kernels to train each of the models. Their findings revealed the prowess of hybrid models in capturing symmetric and asymmetric effects of news on volatility than the classic GARCH, EGARCH, and GJR GARCH methods. In similar veins, Kipkoeh (2014) examined the volatility of the Kenyan shillings (KES) against the United States dollars to analyze the predictive performance of EGARCH models by comparing two distributions: Gaussian and Student's. He used the maximum likelihood estimator and his results revealed that student's distribution provides better performance over other specifications of EGARCH models.

Based on a comprehensive review of the literature relating to modeling and forecasting of exchange rates and GARCH models especially in East Africa and Uganda particularly, it's evident that there exists a very huge gap. To the best of our knowledge, there have been no absolute attempts to model exchange rates using GARCH under any specification. Limited close cases include Etuk

and Natamba (2015) who applied ARIMA to forecast exchange rates and recommended the adoption of the SARIMA (0,1,1) in modeling and forecasting based on the UGX/USD exchange rates in Uganda between August 2014 and February 2015. Namugaya et al. (2014) used daily closing prices between January 2015. They estimated both symmetric and asymmetric GARCH models to examine specific stylized facts such as volatility clustering and leverage. They employed a quasi maximum likelihood method for estimation and AIC and BIC for model selection. Their results indicated that the GARCH (1,1) model outperformed the other competing models in modeling volatility while EGARCH (1,1) performed best in forecasting volatility of USE returns based on MAE and MSE.

The next sections present the models and the materials. Results, discussions, and conclusions are presented in the last three sections of the paper.

## 2. MATERIALS AND METHODS

This section discusses data, mean equation specification, the GARCH family models, distributions densities and forecast evaluation methodologies.

### 2.1. Data

The data used in this study consists of daily foreign exchange rate series of the US Dollar (USD) against the Uganda shillings (UGX) ranging from September 05, 2005 to August 30, 2018, constituting a total of 3738 observations. The data was sourced from the official website of the Bank of Uganda under the statistics section ([www.bou.or.ug/bou/rates\\_statistics/statistics.html](http://www.bou.or.ug/bou/rates_statistics/statistics.html)).

Data transformation was performed to obtain log-returns of the exchange rates series to overcome the difficulties of modeling with non-stationary data in time series. The following formula was used to obtain log returns:

$$r_t = \log\left(\frac{\text{USD/UGX}}{\text{USD/UGX}(-1)}\right) \times 100$$

Where USD/UGX is the daily observation for the USD against UGX while USD/UGX(-1) is the lag of the same on day t. Table 1 in the appendices illustrates the line plots of the two series. It can be observed that the USD/UGX series has a trend suggesting that it's nonstationary while that of the return series reverts to its constant mean reflecting that it's a stationary process. Additionally, volatility clustering can be deduced from the plot of the return series since it's easily observable that periods of low volatility are followed by periods of low volatility while those of higher volatility are followed by the same over a lengthy period. Figure 1 is a plot of the return series at both levels and first difference.

### 2.2. Mean Equation Specification

To address autocorrelation problems in the models, we first tested four mean equations starting from the constant to AR (4) models to ascertain appropriate models. Two models (the constant and AR [1]) were significant and, therefore, we used these two specifications to model various GARCH family models. The two models take the following functional expressions.

**Table 1: Summary statistics for the USD/UGX returns series**

Observations	3238
Mean	-0.000225
Std. Dev.	0.004874
Skewness	0.362888
Kurtosis	20.79384
Jarque-bera	42788.50***

Source: Authors' calculations

Constant Mean equation:  $r_t = \mu + \varepsilon_t$

AR (1) Mean equation:  $r_t = \mu + \sum_{i=1}^q \phi r_{t-i} + \varepsilon_t$

According to Alexander (1961) and Andersen and Bollerslev (1998) the variance was modeled for the above two models on different GARCH models to test different issues across three different distributions to observe various sensitivities based on different tail distributions and kurtosis assumptions of financial series. Variance equation is given by:

$$\varepsilon_t = \sqrt{h_t} v_t, \text{ where } v_t \sim \text{iid}(0,1)$$

### 2.3. The GARCH Family Models

Volatility is a crucial element of investment whose understanding has very important implications for an economy that aspires to grow. Earlier methods for modeling volatility were always focused on variance and standard deviation, while ignoring conditionality and very important aspects of financial time series data such as leverage effects, heavy-tailed distribution, and volatility among others. In response to these shortcomings, Engle (1982) presented a time-varying model that conditionalizes variances of past innovations called the ARCH. This idea was further improved by Bollerslev (1986) by adding past conditional variance to overcome the huge lags specification of the ARCH model. This model was named generalized ARCH (GARCH). Other models have since been developed to model symmetries and asymmetries to all form the GARCH family discussed below.

#### 2.3.1. ARCH

This model is attributed to Engle (1982) based on a seminal work in which he suggested that time-varying conditional heteroscedasticity be modeled by applying past innovations to estimate variance as follows:

$$h_t = \eta_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$$

where  $\varepsilon_{(t-1)}^2$  represents the ARCH 1 process.

#### 2.3.2. Generalized ARCH

Bollerslev (1986) advanced the ideas of Engle (1982) by suggesting a generalized form of ARCH which overcomes the difficulty of huge lags specifications. Precisely, he proposed a model in which heteroscedasticity is determined by past innovations and past conditional variance as a set of regressors represented as a higher-order ARCH represented as follows:

$$h_t = \eta_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}$$

where  $\varepsilon_{(t-1)}^2$  is the ARCH term while is the GARCH term with restrictions imposed as  $\eta_0 > 0$ ,  $\alpha_i \geq 0$  and  $\beta_i \geq 0$ . The sum of ARCH and GARCH coefficients determines the persistence of shocks  $\alpha_i + \beta_i < 1$  to ensure that  $\varepsilon_t$  is stationary with positive variance.

**2.3.3. Exponential GARCH**

This is an exponential model developed by Nelson (1991) to model asymmetric tendencies in volatility. This model relaxes the non-negativity constraint restrictions placed on Alpha and Beta in the GARCH model. This model takes conditional variance as a function of lagged innovations as illustrated below:

$$\ln(h_t) = \eta_0 + \sum_{i=1}^q \alpha_i \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \sum_{i=1}^q \lambda_i \left( \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right) + \sum_{i=1}^q \beta_i \ln(h_{t-1})$$

In this model, the leverage effect is exponential irrespective of the sign on the coefficients.  $\lambda_i < 0$  stands for negative parameters implying negativity shocks will have higher effects on expected volatility than positive shocks of the same magnitude. Here,  $\lambda_i$ ,  $\alpha_i$  and  $\beta_i$  represent leverage effect, shocks magnitude and persistence, respectively. Nelson and Cao (1992) suggest that this model gives freedom to positive and negative shocks to determine volatility and lets large shocks have a superior influence on volatility.

**2.3.4. Threshold GARCH (TGARCH)**

This model is also known as the GJR-GARCH model named after Glosten et al. (1993). The model introduces the aspects of good news and bad news with different effects on the conditional variance. The model is just an augmentation of the standard GARCH with additional ARCH term conditional on past disturbances.

$$\ln(h_t) = \eta_0 + \sum_{i=1}^q (\alpha_i \varepsilon_{t-1}^2 + \lambda_i \varepsilon_{t-1}^2 I_{t-1}) + \sum_{j=1}^p \beta_j h_{t-j}$$

$\lambda_i$  measures leverage effects and I is a dummy equal to 1 where  $\varepsilon_t$  is negative. The good news is  $\varepsilon_{t-1} > 0$  and bad news is  $\varepsilon_{t-1} < 0$  good news influences conditional variance by  $\alpha_i$  while bad news influences conditional variance by  $\alpha_i + \lambda_i$ . When  $\lambda_i > 0$  bad news increases volatility and it implies an increase in leverage effects. Glosten et al. (1993) notes that if  $\lambda \neq 0$  the bad news impact is asymmetric.

**2.3.5. Power GARCH (PARCH) model**

This model was developed by Ding et al. (1993) to model nonlinearities by asymmetric power ARCH of (p, q) order presented as follows:

$$h_t^\delta = \alpha_0 + \sum_{i=1}^q (\alpha_i |\varepsilon_{t-i}| - \lambda_i \varepsilon_{t-i})^\delta + \sum_{j=1}^p \beta_j h_{t-j}^\delta$$

where  $\alpha_i, \beta_i$  are where  $\alpha_i$  and  $\beta_i$  are the standard ARCH and GARCH parameters.  $\lambda_i$  and  $\delta$  represent leverage effects and power terms, respectively. Restrictions here are that  $\delta > 0$  and  $|\lambda_i| \leq 1$ .

**2.3.6. Integrated GARCH (IGARCH)**

This is a special form of GARCH developed to deal with series that have a unit root. It was first introduced by Engle and Bollerslev (1986). The model integrates the series to achieve stationarity. The parameters of the GARCH are restricted to a sum equal to 1 and the constant is ignored to transform a standard GARCH model into IGARCH.

$$h_t = \alpha_i \varepsilon_{t-1}^2 + (1 - \alpha_i) h_{t-1}$$

Here, additional constraints are  $\{\alpha + (1 - \alpha_i)\} = 1$  and  $0 < \alpha_i < 1$ .

**2.4. Distributions Densities**

In modeling GARCH family models, variance is assumed to be stochastic although there is variance in GARCH. GARCH structures generate heavy-tailed outputs even for returns. Therefore, leptokurtic returns can be compatible with normal standardized errors. This study, therefore, considers these distributions associated with the GARCH family.

**2.4.1. The Gaussian distribution**

The Gaussian distribution is also known as the normal distribution and best represented as follows:

$$L_{\text{gaussian}} = -\frac{1}{2} \sum_{i=1}^T (\ln[2\pi] + \ln[\sigma_i^2]) + Z_i$$

where T is the number of observations,  $\sigma$  is the standard deviation and  $\pi$  is the constant pi.

**2.4.2. The student's t-distribution**

This is also called the T distribution and it's almost identical to the normal distribution curve, only that it's a bit shorter and fatter. Under this distribution, the log-likelihood is computed as follows;

$$\text{Student's } s_T = \ln \left( \Gamma \left[ \frac{v+1}{2} \right] \right) - \ln \Gamma \frac{v}{2} - \frac{1}{2} \ln(\pi[v-2]) - \frac{1}{2} \sum_{i=1}^T (\ln \sigma_i^2 + [1+v] \ln \left[ 1 + \frac{z_i^2}{v-2} \right])$$

**2.4.3. The generalized error distribution**

This is a generalized form of the normal distribution that has a natural multivariate form with an unbounded top parametric kurtosis. It has cases similar to the Gaussian and character which controls for kurtosis. This can be represented as follows:

$$L_{\text{GED}} = \sum_{i=1}^T (\ln \left[ \frac{v}{\lambda_v} \right] - 0.5 \left[ \frac{Z_i}{\lambda_v} \right]^v - [1+v^{-1}] \ln 2 - \ln \Gamma \left[ \frac{1}{v} \right] + 0.5 \ln [\sigma_i^2])$$

Where

$$\lambda_v = \sqrt{\frac{\Gamma(1/v 2^{-2/v})}{\Gamma(3/v)}}$$

**2.5. Forecast Evaluation Methodologies**

Forecast accuracy in this study is evaluated based on the mean square error (MSE), root mean square error (RMSE), mean absolute percentage error (MAPE), Thiel's U<sub>1</sub> and Thiel's U<sub>2</sub> statistics and models are selected based on information criteria.

**2.5.1. MAE**

It measures the deviation from the original values. The closer the MAE to zero, the better the goodness of fit and thus the better the forecast. MAE is represented by the following equation.

$$MAE = \frac{1}{n} \sum_{t=1}^n |e_t|$$

**2.5.2. MSE**

The closer the MSE of a model to zero, the preferable the model. MSE takes the following representation:

$$MSE = \frac{1}{n} \sum_{t=1}^n e_t^2$$

**2.5.3. RMSE**

This has similar interpretations as the MSE in the choice of the most preferable model.

$$\sqrt{MSE}$$

**2.5.4. MAPE**

This criterion is a relative measure of MAE which provides relatives performances of different forecast items. The lower the percentage MAPE, the better the forecast model.

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|e_t|}{y_t} \times 100\%$$

**2.5.5. Theil's U**

Thiel's U is a normalized measure of forecast accuracy. There are two types of this Thiel's U. The first (U1) is a measure of forecast accuracy (Thiel, 1958. p. 31-42); The second (U2) is a measure of forecast quality.

$$U_1 = \frac{[\sum_{i=1}^n (P_i - A_i)^2]^{1/2}}{[\sum_{i=1}^n A_i^2]^{1/2}}$$

$$U_2 = \frac{[\frac{1}{n} \sum_{i=1}^n (A_i - P_i)^2]^{\frac{1}{2}}}{[\frac{1}{n} \sum_{i=1}^n A_i^2]^{\frac{1}{2}} + [\frac{1}{n} \sum_{i=1}^n P_i^2]^{\frac{1}{2}}}$$

Where  $A_i$  represents the actual observations and  $P_i$  is the corresponding predictions for the case of  $U_1$  whereas in Thiel  $U_2$  they represent proposed  $U_2$  a pair of predicted and observed changes. Perfect forecasts are those with  $U_1$  closer to the 0 bound while worst forecasts are those closer to 1.  $U_2$  can be interpreted as the RMSE of the proposed forecasting model divided by the RMSE of a no-change model.  $U_2$  values lower than 1.0 show an improvement over the simple no-change forecast.

**3. RESULTS**

This section presents the description of the data and results obtained from the estimation of the various GARCH models.

**Table 2: Unit roots for the return series**

Tests at different levels	ADF test			PP test		
	t-stat.	P-value	Lag	t-stat.	P-value	Lag
Intercept	-38.04378	0.0000***	1	-37.91004	0.0000***	1
Intercept and trend	-38.04626	0.0000***	1	-37.90714	0.0000***	1
None	-37.96551	0.0000***	1	-38.15968	0.0000***	1

Source: Authors' calculations

**3.1. Descriptive Statistics of the USD/UGX Return Series**

Descriptive statistics of the series provide a general glimpse into the behavior of the return series. The following summary statistics are obtained from the series.

Table 1 above provides descriptive statistics for the USD/UGX return series from which a negative mean can be observed, reflecting the depreciation of the exchange rates over time. The returns exhibit a non-symmetric distribution since they are positively skewed thus indicating a depreciation of the Shillings. A look at the coefficient of kurtosis is suggestive of the leptokurtic (fat-tail) distribution of the returns series. The series doesn't also follow a normal distribution as justified by a Jarqua-Bera test which rejects normality assumption at a 1% level of significance. The standard deviation is positive, indicative of high fluctuations in the exchange market.

A further graphical examination of the distribution characteristics of the return series using Q-Q and histogram as in Figure 2 in the appendices shows that observations are scattered in an S-shape pattern far away from the 45-degree line confirming Ahmed and Suliman (2011) who argue that such a scatter pattern is evidence for non-normal distribution in series, which is also confirmed by the histogram below the Q-Q plot. An examination of the incidence of serial correlation shows that the series is autocorrelated except at the first and second lags.

**3.2. Stationarity**

Unit root analysis is a prerequisite in time series modeling and as such, preliminary unit root tests were examined on the USD/UGX series. The series exhibited trend just from the basic graphical examination and we couldn't reject the null hypothesis from the ADF test of unit root implying that the series wasn't stationary. Following these results, we obtained returns from the USD/UGX through a log transformation of the first lag in a process explained above. Displayed below in Table 2 are results of the ADF unit root tests.

**3.3. Tests for Heteroscedasticity: ARCH Effects**

Modeling GARCH models requires securing certainty over the presence or absence of the ARCH effects. This can be effectively executed through the Lagrange multiplier test for ARCH (Engle, 1982). The LM proposes that given  $\varepsilon_t = r_t - \mu$  as the residual for the mean equation  $\varepsilon_t^2$  is then used to test conditional heteroscedasticity, known as the ARCH-LM effect. The AR (1) model for the conditional mean was estimated and the ARCH-LM test was done for the first lag. From the results of the test in Table 3 below, the null hypothesis of No ARCH effects is rejected at 95% confidence interval. This indicates the presence of ARCH effects implying that there is fluctuating variance in the return series.

**Table 3: Estimation of different conditional means and testing ARCH effect**

Variable	(1)	(2)
Dependent variables	Return c	Returns c returns (-1)
C	-0.000225*** (8.57E-05)	-0.000142** (8.01E-05)
AR (1)		0.357624*** (0.016414)
ARCH effects		
Constant	1.70E-05*** (1.83E-06)	1.66E-05*** (1.71E-06)
Lag error squared	0.282662*** (0.016865)	0.199036*** (0.017233)
Ho: No. Arch effect		
F-stat.	280.9180	133.4015
Prob.	0.0000	0.0000

Source: Authors' calculation

### 3.4. Results of Estimation of GARCH Models

The results reported in Tables 4-6 below are estimates of intercept, variance equations and diagnostics of both symmetric and asymmetric GARCH models under Gaussian, Student's T and GED distributions. The tables report estimations for the two mean equations: The intercept (1) and the AR (1). Table 7 above presents the volatility forecast performance of the models across distributions on the basis of information criteria using the whole sample. Additionally, Table 8, shows the forecast accuracy of competing models examined using out-sample data between the periods September 01, 2005 and June 029, 2018.

## 4. DISCUSSIONS

Having established the presence of volatility from the ARCH-LM test, initial GARCH models were estimated to model variance dynamics and asymmetric effects of volatility through GARCH (1,1) and PARCH (1,1), respectively under normal distribution, as reported in Table 4. The results of GARCH (1,1) in the mean equation indicate that the intercept  $\mu$  for both equations is significant at 1%. The AR (1) coefficient of the dependent variable ( $\varphi$ ) is also significant at 1%. Volatility equation for the GARCH (1,1) reveals that the constant ( $\eta$ ), ARCH ( $\alpha$ ) and GARCH ( $\beta$ ) terms are all positive and statistically significant at 1%, with the sums of  $\alpha$  and  $\beta$  exceeding 1, implying indefinite variance due to non-stationarity in the residuals. Diagnostic tests for serial correlation using the Ljung-Box Q-statistics for standardized residuals (Q1) and their squared values (Q2) on the 4<sup>th</sup> and 8<sup>th</sup> lags as advanced by Tse (1998) revealed the presence of autocorrelation as Q1 statistics were significant at 5% level.

Although the AR (1) equation provides evidence of no ARCH effects, the F-statistic for first the mean equation was significant at 5% justifying the presence of ARCH effects. Given the shortcomings of the Gaussian distribution manifested by skewness and excess kurtosis of return series as shown in Figure 2 in the appendices, other distribution assumptions were tested. Specifically, student's t and GED were examined following Bollerslev (1987). The results under normal distribution are similar to the results under the assumptions of T and GED distributions except that the ARCH effect is eliminated under the two distributions. However, the

problem of autocorrelation is persistent as reported in the GARCH (1,1) models in Tables 5 and 6.

The results of PARCH (1,1) tabulated in Table 4 indicate that AR (1) is positive and significant and so are ARCH and GARCH coefficients at 1% levels which reflect the responsiveness and persistence to shocks, respectively. In this model,  $\alpha$  captures the response of conditional volatility to appreciation or depreciation of the Ugandan shillings while  $\beta$  captures its persistence to market shocks. Since their sum goes beyond 1, evidence of an infinite nonstationary variance is established. The leverage coefficient ( $\lambda$ ) is positive and significant implying a higher influence of negative past innovations on volatility than positive values of the similar magnitudes on conditional variance. The results of a negative coefficient on leverage would produce interpretations that are vice versa to the ones realized above. These results established that appreciation or depreciation of the Ugandan shillings against the dollar doesn't have a directional effect on volatility. However, the model is not a standard GARCH model given the positive value and significance of  $\delta$ , which is  $<2$  in the model, Ding et al. (1993). The statistics for the diagnostics indicate the presence of autocorrelation and the absence of ARCH effects as shown by significant Q1 statistics and insignificant F-statistic. Similar findings are realized when the distribution is changed to student's T or GED under PARCH (1,1) in Tables 5 and 6 respectively. Under T distribution, ARCH effects are no longer a problem although serial autocorrelation continues to affect the model as manifested by highly significant Q statistics. However, better results are observed under the GED assumption, in which both autocorrelation and ARCH effects are eliminated in the AR (1) mean equation.

Under the EGARCH (1,1) model in Table 4 with the assumption of the normal distribution, the AR (1) coefficient is positive and significant just like  $\alpha$  and  $\beta$  coefficients which are the asymmetric and size parameters, respectively. This model considers nonnegative restrictions placed on variance. The asymmetric term ( $\lambda$ ) is negative and significant at 5% revealing the presence of asymmetric effects implying that negative shocks to the Ugandan Shillings will have a higher influence on expected volatility than positive shocks of the same magnitude. The coefficients for the intercept and AR (1) mean equations are significant at 1%. There are no ARCH effects although there is evidence of autocorrelation in the residuals as indicated by very significant Q statistics. A change of distribution to student's t for the EGARCH (1,1) as reported in Table 5 provides no improvement either in terms of diagnostics, although both mean, and variance coefficients remain similar. However, a further change of distribution to GED in Table 6 deteriorates the model. Both the ARCH and GARCH parameters remain positive and significant, but the asymmetric parameter becomes insignificant, although it's negative.

Starting with the Gaussian distribution assumption in Table 4, the results of TARARCH (1,1) provide a highly significant level for the AR (1) coefficient in the mean equation. In the variance equation, The ARCH ( $\alpha$ ) and GARCH ( $\beta$ ) coefficients are also largely significant. The asymmetric parameter ( $\lambda$ ) is negative and significant with no evidence of ARCH effects

**Table 4: Estimations of GARCH family models under Gaussian distribution**

Parameters	GARCH		EGARCH		IGARCH		TARCH		PARCH	
	1	2	1	2	1	2	1	2	1	2
$\mu$	-0.000102***	-8.43E-05***	-5.66E-05***	-7.38E-05***	-0.000242***	-0.000174***	-0.000112***	-0.000102***	-5.97E-05	7.92E-05***
	3.05E-05	2.60E-05	2.44E-05	-2.62E-05	1.23E-05	1.04E-05	3.65E-05	3.15E-05	3.02E-05	3.00E-05
		0.398706***		0.400208***		0.392804***		0.400919***		0.395934***
$\varphi$		6.01E-09		0.013949		0.010348		0.014245		0.013161
<b>Variance equation</b>										
$\eta$	1.29E-07***	4.69E-08***	-0.950869***	-0.835380***			1.28E-07***	4.37E-08***	3.08E-05***	3.03E-05***
	9.54E-09	0.014443	0.038413	0.036404			9.58E-09	5.94E-09	1.27E-05	1.44E-05
$\alpha$	0.376879***	0.315093***	0.536711***	0.506496***	0.139184***	0.127316***	0.358307***	0.271238***	0.337945***	0.284962***
	0.011893	0.007609	0.012467	0.010119	0.002021	0.002012	0.018045	0.012732	0.012073	0.008936
$\lambda$			-0.015890**	-0.031287***			0.030578	0.066002***	0.026995**	0.068309***
			0.008182	(0.008239)			0.022196	0.018015	0.016621	0.018716
$\delta$									0.747058*	0.794211***
									0.007401	0.005686
$\beta$	0.724456***	0.775484***	0.949741***	0.958378***			0.725986***	0.781830***	1.207540***	1.115557***
	0.006340	0.003784	0.002805	0.002758			0.006539	0.003931	0.061608	0.070556
1- $\alpha$					0.860816***	0.872684***				
					0.002021	0.002012				
<b>Diagnostics</b>										
Q1(4)	422.59***	47.076***	414.90***	41.671***	545.45***	64.827***	423.19***	46.648***	419.05	44.432***
Q2(8)	458.16***	62.287***	445.94***	54.156***	582.84***	79.819***	448.42***	61.184***	452.53	57.858***
Q1(4)	8.4613***	4.0447	9.3195**	5.6006	39.314***	16.786***	1.0063	4.4682	10.852	7.3788
Q2(8)	12.989***	9.2877	13.705***	11.390	42.779***	19.498***	1.8773	9.9280	14.256	12.202
LLK	14021.22	14221.47	14058.89	14265.72	14058.89	14086.35	195.4121	14224.45	14036.95	14239.77
F stat.	3.211059	1.962559	4.450274	2.421528	39.04113	14.85014	0.019050	2.055045	0.944095	4.804863
Prob.	0.0732	0.161336	0.0350	0.119776	0.0000	0.000119	0.8909	0.151799	0.336790	0.028451

1 and 2 represent the first and second mean equations, respectively. On the other hand, \*\*\*, \*\* and \* indicate significance at 1%, 5% and 10% levels, respectively

Table 5: Estimations of GARCH family models under student's T distribution

Parameters	GARCH		EGARCH		IGARCH		TARCH		PARCH	
	1	2	1	2	1	2	1	2	1	2
$\mu$	-1.91E-05	-1.58E-05	-2.33E-05	2.78E-05	-9.25E-06	1.71E-05	-2.10E-05	-2.08E-05	-2.47E-05***	-2.31E-05***
	2.03E-05	1.88E-05	2.27E-05	2.09E-05	4.48E-05	1.57E-05	2.08E-05	1.91E-05	2.23E-05	2.01E-05
	0.393164***	0.393164***	0.393164***	0.393164***	0.393164***	0.393164***	0.397789***	0.394403***	0.397789***	0.390179***
	0.015968	0.015968	0.015968	0.015968	0.015968	0.015968	0.013094	0.015875	0.013094	0.015591
$\eta$	3.37E-08***	1.73E-08***	-1.031161***	-0.782068***			3.33E-08***	1.65E-08***	8.63E-05	2.03E-06
	1.17E-08	7.12E-09	0.082024	0.067923			1.16E-08	6.94E-09	6.58E-05	2.39E-06
	0.780954***	0.620822***	0.735059***	0.640842***	-0.000248***	0.179547***	0.708341***	0.511020***	0.492357***	0.470335***
	0.110541	0.093901	0.049095	0.048374	1.42E-05	0.007095	0.107410	0.085020	0.048577	0.058621
$\lambda$			-0.039806**	-0.053556***			0.111542	0.154882**	0.050256	0.076312***
			0.021815	0.022998			0.081354	0.076833	0.032041	0.035901
$\delta$									0.697083***	0.749793***
									0.015599	0.015120
$\beta$	0.644117***	0.708269***	0.950712***	0.966872***			0.647639***	0.716939***	1.001369***	1.395130***
	0.014521	0.014091	0.006303	0.005097			0.014484	0.013931	0.110981	0.159095
1- $\alpha$					1.000248***	0.820453***				
					1.42E-05	0.0070595				
<b>Diagnostics</b>										
Q1(4)	353.88***	50.866***	370.65***	370.65***	438.72***	61.333***	353.84***	49.997***	361.75***	50.895***
Q2(8)	385.45***	65.202***	399.51***	399.51***	449.29***	75.331***	384.90***	63.469***	392.62***	64.126***
Q1(4)	2.8225	2.8428	6.0577***	45.706***	659.70***	8.3659**	2.5957	2.8632	3.0180	2.8503
Q2(8)	4.3051	6.3962	9.9458***	57.915***	822.79***	14.316**	3.9807	6.5203	4.4438	6.6199
LLK	14337.35	14569.37	14351.29	14580.91	13583.71	14502.41	14338.39	14571.83	14348.21	14574.99
F stat.	0.001667	0.006002	0.184422	0.370255	244.6195	7.343403	0.001469	0.015467	0.400793	0.292373
Prob.	0.9674	0.9383	0.667629	0.5428	0.000000	0.0068	0.969424	0.901032	0.526725	0.588741

Source: Authors' calculations. 1 and 2 represent the first and second mean equations, respectively. On the other hand, \*\*\*, \*\*, \* and \* indicate significance at 1%, 5% and 10% levels, respectively

Table 6: Estimations of GARCH family models under GED distribution

Parameters	GARCH		EGARCH		IGARCH		TGARCH		PARCH	
	1	2	1	2	1	2	1	2	1	2
$\mu$	-1.11E-11	-9.41E-07	-2.75E-06	-3.22E-06	-2.78E-06	-3.80E-06	-2.75E-06	-3.22E-06	2.23E-15	-3.21E-06
	1.31E-05	1.53E-05	1.65E-05	1.67E-05	1.33E-05	1.19E-05	1.56E-05	1.55E-05	1.22E-05	1.61E
		0.386534***		0.385706***		-0.388213***		0.385531***		0.384398***
$\varphi$		0.014026		0.014018		0.011831		0.014002		0.013750
<b>Variance equation</b>										
$\eta$	3.83E-08***	2.16E-08***	-1.027212***	-0.826357***			3.81E-08***	2.01E-08	5.12E-05	8.12E-06
	1.10E-08	6.66E-09	0.087902	0.073586			1.09E-08	6.48E-09	4.48E-05	9.10E-06
	0.556302***	0.427962***	0.660658***	0.569740***	0.180368***	0.160132***	0.520853***	0.369534	0.423816***	0.357691***
$\lambda$	0.046206	0.033710	0.034928	0.030372	0.006460	0.006178	0.056031	0.041026	0.035286	0.027671
			-0.026899	-0.035270***			0.056365	0.086660***	0.036938	0.061937
$\delta$			0.021629	0.020794			0.064529	0.053602	0.034860	0.038192***
$\beta$	0.664658***	0.723195***	0.950623***	0.963362***			0.666799***	0.729562***	1.068744***	1.230320
	0.015506	0.012996	0.006528	0.005386	0.819632***	0.839868***	0.015518	0.012811	0.125338	0.153480
$1-\alpha$					0.006460	0.006178				
<b>Diagnostics</b>										
Q1(4)	372.15***	52.882***	377.32***	48.237***	508.72***	67.175***	371.93***	53.137***	379.42	51.792***
Q2(8)	404.83***	68.026***	406.66***	61.066**	544.15***	82.016***	404.12***	67.547***	411.20	65.594***
Q1(4)	0.458	3.0450	6.5281	4.9009	24.370***	11.387***	3.4715	3.1485	4.2372	3.5061
Q2(8)	0.680	7.1814	10.793	11.595	30.984***	16.513***	5.4727	7.3877	6.2422	7.7638
LLK	14367.00	14573.01	14382.05	14588.06	14293.68	14513.56	14367.45	14574.53	14376.80	14580.39
F stat.	0.054687	0.067899	0.400706	0.504495	22.25495	10.59354	0.052041	0.093782	0.925759	0.860690
Prob.	0.815115	0.7944	0.526769	0.477583	0.000002	0.0012	0.8195	0.7594	0.336039	0.3536

Source: Authors' calculations. 1 and 2 represent the first and second mean equations, respectively. On the other hand, \*\*\*, \*\*, \* and \* indicate significance at 1%, 5% and 10% levels, respectively

**Table 7: Forecast models evaluation based on information criteria**

Criteria	GARCH	TARCH	EGARCH	IGARCH	PARCH
<b>Normal distribution</b>					
LLK	14221.47	14224.45	14265.72	14086.35	14239.77
AIC	-8.783735	-8.784954	-8.810456	-8.701483	-8.79380
SBC	-8.774340	-8.773680	-8.799182	-8.695846	-8.780649
<b>Student t-distribution</b>					
LLK	14569.37	14571.83	14580.91	14502.41	14574.99
AIC	-8.998067	-8.998968	-9.004578	-8.957931	-9.000301
SBC	-8.986793	-8.985815	-8.991425	-8.950415	-8.985269
<b>GED distribution</b>					
LLK	14573.01	14574.53	14588.06	14513.56	14580.39
AIC	-9.000316	-9.000634	-9.008998	-8.964817	-9.003637
SBC	-8.989042	-8.987481	-8.995844	-8.957301	-8.988605

Source: Authors' calculations

**Table 8: Forecast evaluation for GARCH family models based symmetric lost functions**

Measures of Accuracy	GARCH	PARCH	EGARCH	TARCH	IGARCH
<b>Normal distribution</b>					
RMSE	0.004299	0.004302	0.004290	0.004302	0.004327
MAE	0.002973	0.002976	0.002964	0.002976	0.003014
MAPE	95.97247	95.51312	97.78375	95.51312	101.4370
Theil's U1	0.979852	0.977650	0.988710	0.977650	0.957424
Thiel's U2	1.008709	1.009915	1.004672	1.009915	1.025187
<b>Student's T distribution</b>					
RMSE	0.004283	0.004283	0.004283	0.004283	0.004281
MAE	0.002956	0.002957	0.002957	0.002957	0.002954
MAPE	99.23984	99.07129	99.07102	99.15368	99.64783
Theil's U1	0.996071	0.995208	0.995307	0.995629	0.998172
Thiel's U2	1.002243	1.002487	1.002487	1.002366	1.001694
<b>GED distribution</b>					
RMSE	0.004279	0.004280	0.004280	0.004280	0.004280
MAE	0.002953	0.002953	0.002953	0.002953	0.002956
MAPE	100.0000	99.89787	99.89788	99.89788	99.89697
Theil's U1	1.000000	0.999469	0.999469	0.999469	0.999464
Thiel's U2	1.001267	1.001387	1.001387	1.001387	1.001388

Source: Author's calculations

further establishing the evidence for asymmetric effects. The model, however, suffers from autocorrelation. Unlike results from the Gaussian assumption, the asymmetric parameter is insignificant in the AR (1) type mean equation, the parameter is positive under both mean equations although diagnostics don't support the model when distribution assumption is made as a student's t or GED as can be seen in the TARCH (1,1) tabulated in Tables 5 and 6 respectively. No ARCH effects, but autocorrelation persists. Variance also continues not to be well behaved given the fact that the persistence parameters are still higher than the unitary value.

The autoregressive coefficient for the lag dependent variable under IGARCH (1,1) indicates a positive and significant parameter  $\phi$  from the results under normal distribution in Table 4. This model imposes the persistence parameter, to sum up to unit restrictions on the model. Here, the restrictions placed on the model are positive and significant. However, they don't overcome the diagnostics problems of autocorrelation and ARCH effects established by

significant Q and F statistics. Alteration of distributions doesn't change anything either since similar results are obtained under both T and GED distributions as observed from the IGARCH (1,1) results in Tables 5 and 6 respectively.

Table 7 above presents the volatility forecast performance of the models across distributions compared based on information criteria using the whole sample. Log-likelihood (LLK), Schwartz Bayesian (SBC) and Akaike information criteria (AIC) are used. It can be observed that although student's t distribution out-performed Gaussian, the general performance of the models improved when GED distribution was used since the LLK increased while AIC and SBC decreased. Specifically, an examination of the models under GED distribution reveals that EGARCH (1,1) provided the best fit among the models used.

Forecast accuracy of competing models was examined using out-sample data between the periods September 01, 2005 and June 29, 2018 as presented in Table 8 and Figure 3. The models were estimated and used to make a month ahead forecast of exchange rates starting from July 02, 2018 to August 30, 2018. The performance was compared based on symmetric lost functions which include: RMSE, MAE and Thiel's inequality. A comparison was made across distributions and as it can be observed, similar observations to comparisons based on symmetric lost functions can be made. Better forecasts are realized when student's t and GED are used. However, GED provides much better results with lesser values of RMSE, MAPE and Thiel's U. All models produced very slightly superior performances overall in different criteria. However, overly; EGARCH (1,1) and GARCH (1,1) have shown plausible consistency in minimizing RMSE, MAE and providing better fit based on Thiel's U.

## 5. CONCLUSION

This study drew motivations from the knowledge that volatility has important economic implications for international investment, risk management, remittances, and stock pricing among others. This is particularly relevant for developing economies such as Uganda seeking to expand their share of international trade and realize stable trade balances. We model volatility using GARCH family models by taking into consideration the assumptions on distribution

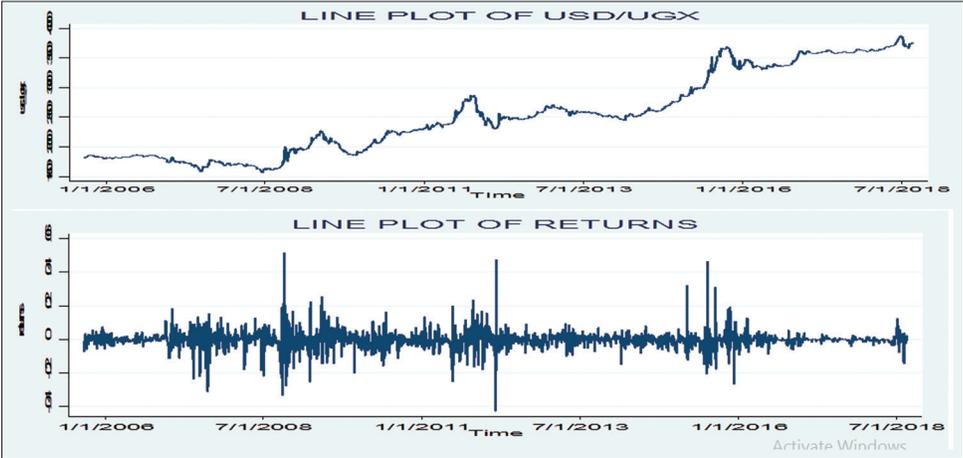
densities and making forecasts to determine appropriate models. Findings established various stylized facts of returns such as volatility clustering, leverage and leptokurtic nature of the series. Specifically, although all models showed phenomenal performance in establishing symmetries (GARCH [1,1] and asymmetries PARCH [1,1], EGARCH [1,1]), TGARCH (1,1); only PARCH (1,1) under the student's t distribution assumption was able to overcome diagnostics issues of autocorrelation and ARCH effects. Since the leverage coefficient ( $\lambda$ ) was positive and significant in PARCH (1,1), it could easily be established that negative past innovations influenced volatility than positive values of similar magnitudes. This implies that the appreciation or depreciation of the Ugandan shillings against the dollar didn't have directional effect on volatility. Coefficients capturing responsiveness and persistence provided evidence that the Ugandan shillings was responsive to shocks and that conditional variance is most likely to persist. This is demonstrated by conditional variance in Figure 4. In modeling volatility, student's t distribution provided better performance since loglikelihood increased compared to Gaussian and GED distributions. On the other hand, forecasting was done under two strands: the entire sample and in-sample strands evaluated based on information criteria and symmetric lost function criteria respectively. EGARCH (1,1) provided the best goodness of fit when AIC, SBC, and LLK were used because it produced smaller AIC and SBC values and higher LLK values compared to other models. All models were very competitive when symmetric lost functions were used, however, GARCH (1,1) and EGARCH (1,1) produced slightly better results based on RMSE, MAPE, MSE and Thiel's U which were on average observed to be slightly lower. All models improved when distribution was changed to student's t or GED, although GED provided the overall best performance in forecasting. Useful conclusions are that the volatility in the Ugandan shillings is very responsive to shocks (explosive) which have been established to be persistent and whose forecast is expected to persist. Countermeasures are advised through macroeconomic and fiscal adjustments in addition to enhancing a stable political environment. Finally, this study provides a ground for subsequent studies to approach GARCH volatility modeling using other methods such as non-parametric Bayesian methods in machine learning and neural networks. Plausible contributions will have been made should future studies consider stock markets and new variables such as inflation and interest rates across distributions in a comprehensive scope of both symmetric and asymmetric GARCH models, and in a panel of several markets/countries.

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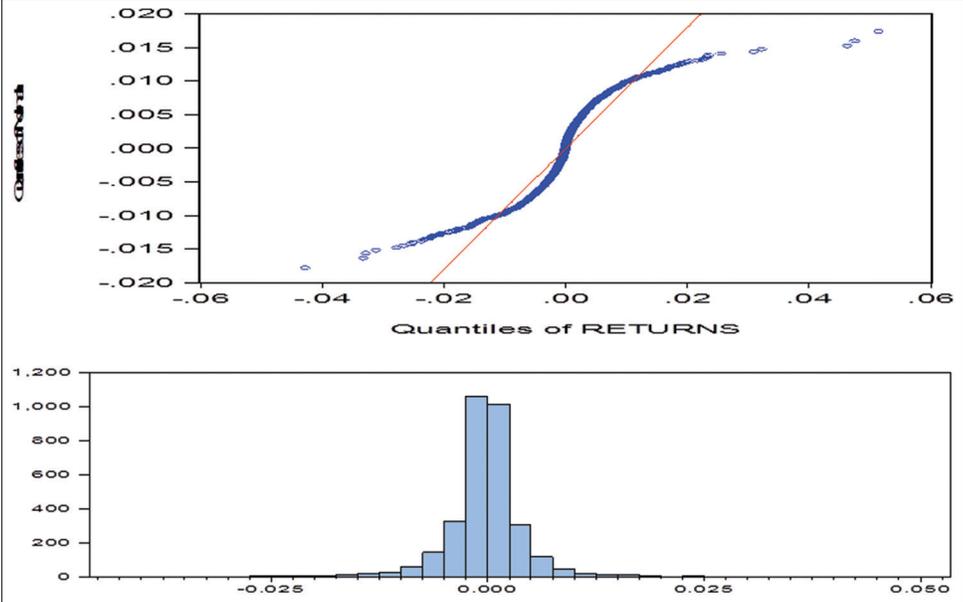
### APPENDICES

Figure 1: Line plots of USD/UGX and return series



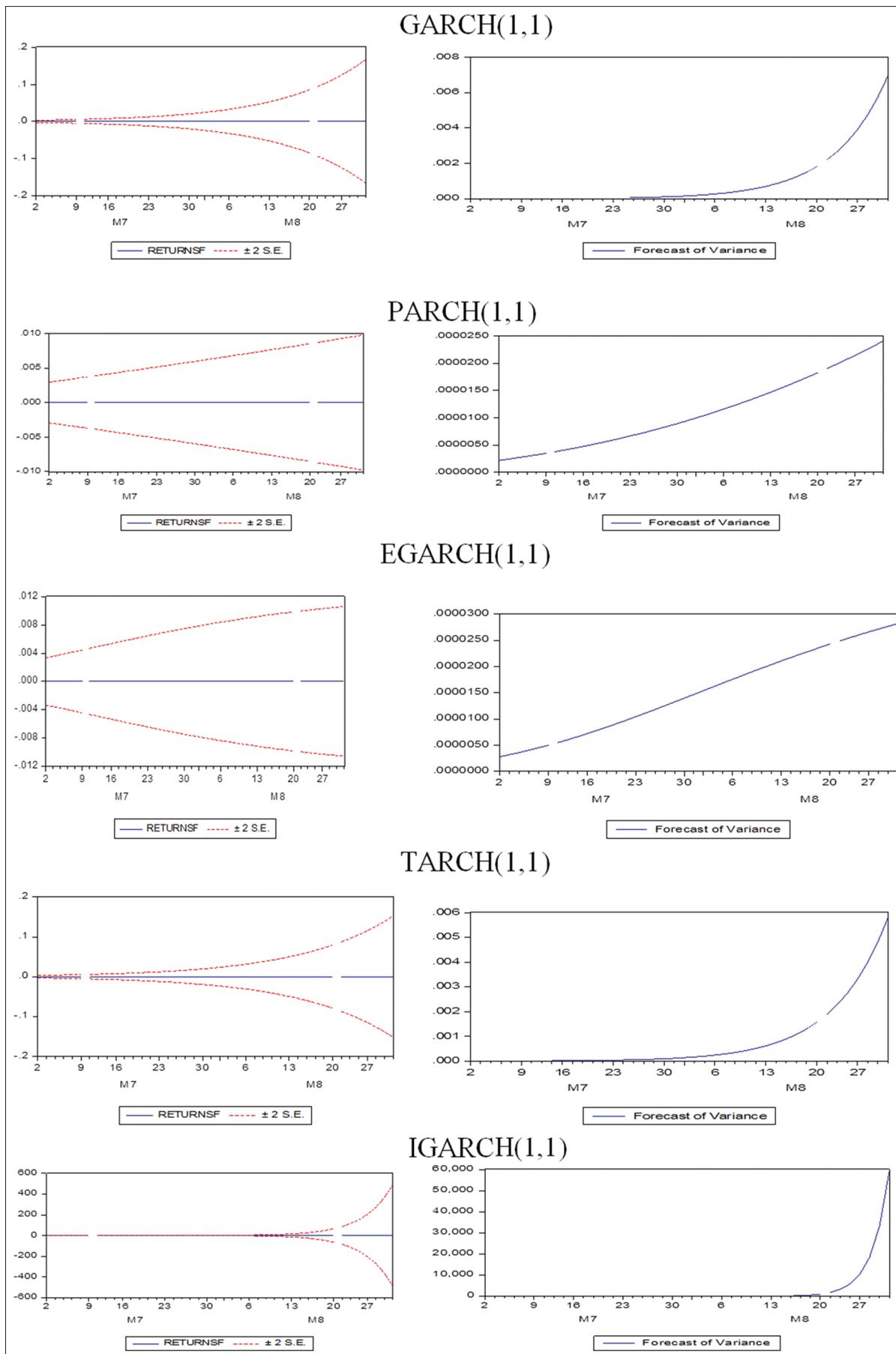
Source: Authors' illustrations

Figure 2: Empirical quantiles and histogram tests for normality



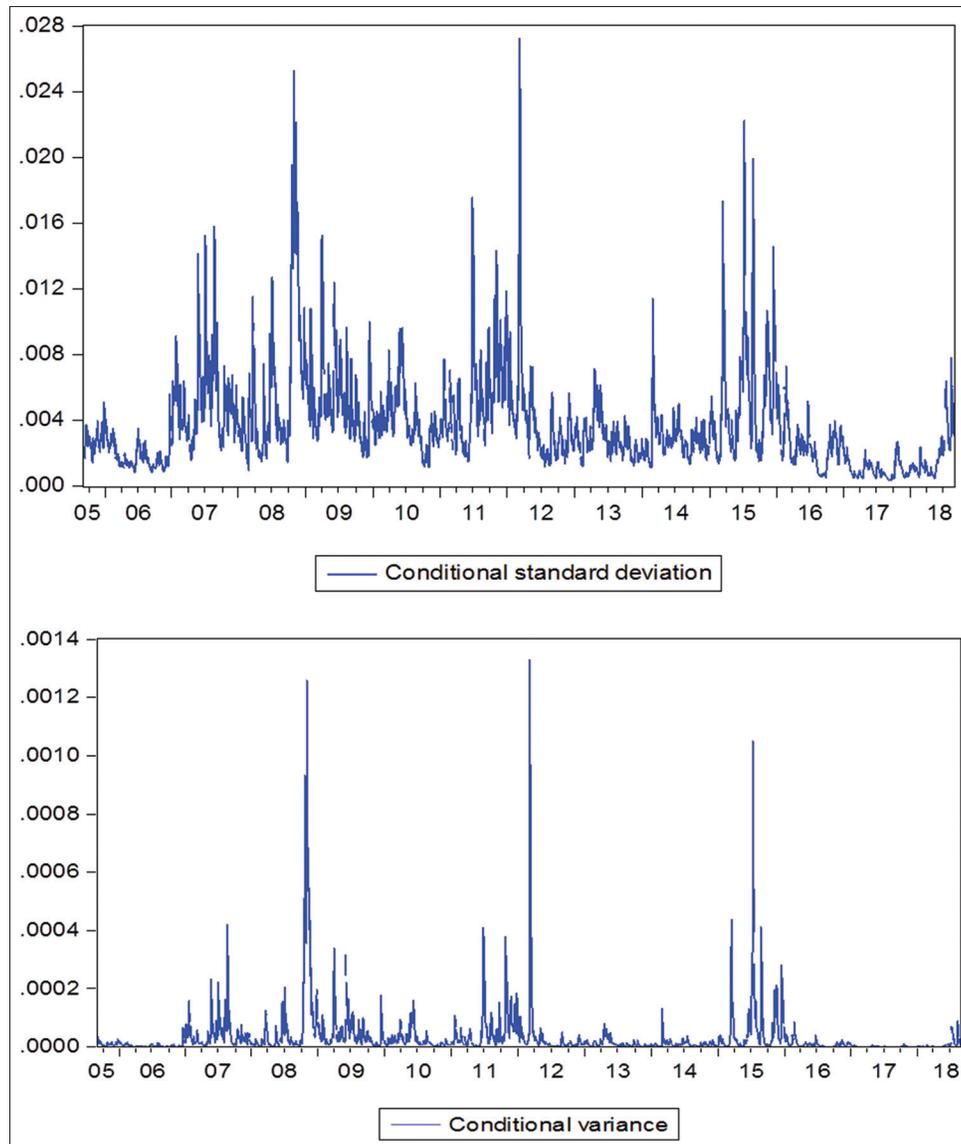
Source: Authors' illustrations

**Figure 3:** Returns and variance forecast graphs for the GARCH family models



Source: Authors' illustrations

**Figure 4:** Conditional variance and standard deviation for the return series



Source: Authors' illustrations